The Unintended Consequences of a Global Carbon Tax
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Abstract

Human activities are putting greater pressure on the climate system, on biological integrity, on biogeochemical systems, and many other earth system processes. There is demand for ambitious policy interventions to reduce each one of these pressures, but the challenge for the international community may be even more difficult than just trying to ‘apply the brakes.’ The problem is rather more like parallel parking, where backing away from one obstacle may put us on a collision course with another. Staying within a ‘safe operating space’ for humanity requires that, as we take steps to reduce the pressure on one earth system process, we do not inadvertently increase the pressure on others. In this paper, we present a novel stylized multi-sector general equilibrium model with multiple earth system processes, and calibrate it with data from the Global Trade Analysis Project (GTAP) to study the global consequences of key policy proposals. Our main finding is that a global carbon tax, aside from reducing carbon emissions, would have many unexpected benefits, single-handedly easing pressure on nearly all of the critical earth system processes. It however also carries with it an increased demand for land in agriculture and further pressure on fisheries, such that supplementary policies may be needed as a safeguard.

1 Introduction

The Earth has been in a remarkably stable state over the last 10,000 years, but human activities on the planet since the industrial revolution are now starting to threaten this balance. Rockström et al. (2009) developed a list of nine key Earth system processes (ESPs) that are essential to maintaining stability, and nine corresponding ‘planetary boundaries’ within which human development could proceed without risking unacceptable global environmental change (Steffen et al., 2015). But going beyond these boundaries could have serious destabilizing effects, with potentially catastrophic consequences for human civilization.

The nine ESPs include many different aspects of our environment, and are affected by a wide range of human activities. They are: Climate change, Ocean acidification, Biodiversity loss, Biogeochemical processes (Nitrogen and Phosphorous loading), Land system change, Freshwater use, Stratospheric ozone depletion, Atmospheric aerosol loading, and Novel entities (chemical
pollution). This list identifies key areas that require attention from policymakers, but at the same time sets up a seemingly un navigable obstacle course for them. Staying within a ‘safe operating space’ for humanity requires that our treaty on biodiversity loss does not undermine our efforts to combat climate change, and vice versa, raised to the ninth power. Even though the planetary boundaries have now become a central conceptual framework shaping discussions of our interaction with the planet, we lack a clear understanding of how key policies affect our movements in this nine-dimensional space.

In this paper we present a stylized general equilibrium model built to explore the consequences of global environmental policies in a world with many planetary boundaries. Our goal is to provide a transparent framework that lets us discover and study the complex and sometimes unexpected ways in which policies can interact with multiple ESPs, and with each other. In so doing, we bring together several strands of research. Researchers have often modeled the activities putting pressure on key ESPs in isolation. This includes the substantial work on climate change, on local pollution (including fertilizer use), and on land-use change (particularly in the context of biofuels policy). Research that deals with more than one of these issues is typically based on large-scale computational models or counterfactual simulations, primarily asking questions related to global land-use and deforestation, and the consequences for climate change and food security (see, for instance, Erb et al. 2016; Popp et al. 2014; Rosegrant et al. 2008a; Tilman et al. 2011). The policy implications of considering all nine planetary boundaries are potentially far-reaching (Crépin et al., 2015), but a clear integrated economic assessment of this framework has remained elusive.

Our intuition can be a poor guide when there are multiple linked externalities. For instance, climate change mitigation is often analogised to ‘applying the brakes’ to anthropogenic greenhouse gas emissions. But a policy aimed at reducing emissions could conceivably destabilise the planet by increasing the pressures on other ESPs, such as by encourage greater freshwater use or deforestation. The problem of policy-making with multiple planetary boundaries can be better analogized to parallel parking, where the challenge is to simultaneously respect boundaries on all sides. The safe operating space is the intersection of the regions in which all boundaries are respected. And there is no guarantee that an individual policy that moves us away from one boundary, will not also move us towards another. But this intuition only takes us so far. In the planetary boundaries framework, the topology of the ‘safe operating space’ depends upon the degree to which the ESPs are linked via economic markets, and we need an integrated economic analysis to understand how key policies alter our position in this space.

Our analysis proceeds in four steps. In section 2 we undertake a brief economic accounting of the activities that are putting pressure on these ESPs. In section 3 we give an overview of

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1It is cited repeatedly in the United Nations Environment Programme’s world assessment reports GEO-5 and GEO-6, and the former UN secretary-general Ban Ki-moon endorsed the concept following a report from the High Level Panel on Global Sustainability. The draft document presented for world leaders at the Rio+20 Summit endorsed them. In 2015 the updated findings were presented in the World Economic Forum in Davos. It has also featured in several public media channels such as the New York times, Washington Post and The Economist to name a few.
existing policies. In section 4 we outline a simple model capable of capturing the central stylized facts from the preceding sections, as well as our approach to parameterizing and solving it. In section 5 we discuss our findings.

Two features of our model and analysis are worth highlighting. First, our goal has been to develop a simple model that, while conducive to transparent analysis, nonetheless captures all the key economic linkages needed for a full economic analysis of the planetary boundaries framework. The model describes a world of decentralized endogenous decision making, wherein economic actors make production and consumption choices that, together, influences important markets and thereby one or more of the key ESPs. In this context, a policy attempting to correct the externality related to a single planetary boundary could, through its effects on related markets, affect the ESPs in ways that either ameliorate or exacerbate the pressure on other boundaries (Bennear and Stavins, 2007).

Second, specific modeling decisions have been made to maintain transparency and stay in the spirit of the original PB framework. This includes considering a static general equilibrium framework, with a focus on comparative statics, rather than explicit welfare computation or consideration of socially optimal policies. This is not to say that these aspects are unimportant, only that we leave them for future research.

We largely discuss the nature of the economic responses directly, focusing on the qualitative nature of key economic relationships, such as substitutability and complementarity in production and demand. Consequently, we focus on the direction towards which policies would nudge ESPs i.e. whether policies considered ameliorate or exacerbate the pressure on different ESPs. We use a variety of sources to parameterize the model: sectoral data are drawn from the widely used Global Trade Analysis Project (GTAP) while estimates of key elasticities of substitution are obtained from the relevant literature. Finally, we discuss how and why our qualitative insights are plausible in the short-to-medium-run.

Our main finding is that a higher global carbon tax is expected to single-handedly reduce almost all of the planetary pressures. This follows since fossil fuel usage is a key driver of many of the PB processes such as climate change and ocean acidification. Additionally, other reduced pressures follows, from indirect effects working through the general equilibrium market response to an increased fossil fuel price. For instance, complementarity between nitrogen (the production of which is highly reliant on natural gas) and phosphorous leads to decreased use of phosphorous. The important exceptions are an increased demand for land-use in agriculture (which becomes relatively more competitive as a production factor when the price of fertilizers increases) and fish (the demand for which increases due to decreased production of food from agriculture). We also consider complementing the carbon tax with a land use policy. This avoids the negative effect on land use. The increased pressure on biological integrity from fisheries still remains. Without a more careful analysis of the drivers behind loss of biological integrity we cannot determine whether the decreased pressure from other factors or the increased pressure from fisheries are more crucial. In summary, somewhat surprisingly, we find that a two-policy combination ensures
that the pressure on almost all planetary boundaries are reduced with the possible exception of biological integrity. A caveat to this positive finding, however, is that increases in food prices are likely to result from such combinations of policies, at least in the short-run.

2 Economic drivers of planetary pressures

We turn next to evaluating which economic activities are the principal sources of anthropogenic pressure on the planetary boundaries, looking at each relevant ESP in turn.

**Climate change** is caused by an increased atmospheric concentration of greenhouse gases, and human activities have been identified, with a high degree of confidence, as the primary driver of this increase. Steffen et al. (2015) estimate the planetary boundary to be an atmospheric CO$_2$ concentration of 350 parts per million (ppm). Carbon dioxide concentrations have increased by 40% since pre-industrial times, primarily driven by fossil fuel emissions (30.4 GtCO$_2$/y) and secondarily from emissions related to land use change (3.3 GtCO$_2$/y) (IPCC, 2013). The largest economic sector driving these emissions is energy production which contributes to approximately 35% of total emissions, but also from activities such as manufacturing (31%), transportation (14%) and building (19%) (IPCC, 2013). Another large contributor to emissions is agriculture, mainly due to deforestation, livestock production, and soil and nutrient management. According to the (IPCC, 2013), annual GHG emissions from agricultural production in 2000–2010 were 5.0–5.8 GtCO$_2$eq/yr while the annual GHG flux from land-use and land-use-change activities accounted for approximately 4.3–5.5 GtCO$_2$eq/yr. Hence, apart from regular manufacturing and transport-related activities the agricultural activity stands out as a significant contributor to climate change through both land-conversion-related emissions and production activities per se.

**Biogeochemical flows** of Phosphorous and Nitrogen from soils and into freshwater systems into oceans can cause widespread eutrophication and a large-scale anoxic event. Steffen et al. (2015) estimate that we must apply no more than 6.2 teragram (Tg) of Phosphorous per year to erodible soils, and 62 Tg of Nitrogen. The source of excess Phosphorous and Nitrogen is almost exclusively the agricultural sector. Almost 90% of the global Phosphate rock production is used to make fertilizer (Prudhomme, 2010), and in many regions and watersheds, runoff created by applying synthetic Nitrogen fertilizer in agricultural production dominates the Nitrogen flux (Howarth, 2008). These fertilizers have been instrumental in achieving rapid productivity growth in the past half-century (Byerlee et al., 2014), but as a result the global surplus of Phosphorous has increased from 2 Tg per year to 11 between 1950 and 2000, and Nitrogen increased from 36 Tg per year to 138 (Bouwman et al., 2013). For our purposes here, it is interesting to note that the industrial process for fixating Nitrogen from the air is heavily reliant on natural gas with 33 million British thermal units (mm Btu) of natural gas needed to produce 1 ton of ammonia and natural gas accounting for 72-85% of the production cost (Huang, 2007).

**Ocean acidification** is in large part caused by an increase in the concentration of free H$^+$
ions in the surface ocean, which makes it harder to synthesize the Aragonite that makes up shells and corals. Steffen et al. (2015) estimate that we need to remain above at least 80% of the pre-industrial Aragonite saturation state to prevent serious deterioration. The increase in the concentration of free H$^+$ ions occurs primarily as a consequence of an increased atmospheric CO$_2$ concentration, and in keeping this below 350 ppm we would also stay on the right side of the ocean acidity boundary. It is therefore mainly fossil-fuel-based energy production and agricultural emissions that create pressure on this planetary boundary. But agricultural production contributes to ocean acidification by another mechanism as well. Nutrient runoff can fuel massive algal blooms, which deplete bottom waters of oxygen (O$_2$) and release CO$_2$ when the organic matter from these blooms is respired by bacteria (Sunda and Cai, 2012).

**Freshwater use** reduces natural availability, and can ultimately lead to regime shifts in the functioning of ecosystems that depend on flows from rivers, lakes, and renewable groundwater stores. Preventing this obviously requires management at the basin-level, but taken in aggregate, imprudent management is consistent with global consumption in excess of 4,000 cubic kilometers of freshwater per year (Steffen et al., 2015). Freshwater use is to a very large extent a consequence of agricultural production. Agriculture is the largest consumptive use of freshwater, accounting for an estimated 92% of global freshwater use annually (Hoekstra and Mekonnen, 2012). One fifth of the agricultural sector’s water footprint can be attributed to production for export, implying that local availability is affected by globalization and trade.

**Land-use change**, specifically the conversion of forested land to other uses, can substantially affect the climate by altering evapotranspiration and the albedo of the land surface. Steffen et al. (2015) estimate that we would need to maintain at least 75% of original forest cover, to avoid such disruptive changes. The main driver of land-use change is agriculture. Agriculture is today the largest user of land on the planet (about two fifths) (Foley et al., 2005, 2011). Between 1980 and 2000, the majority of new agricultural land across the tropics came at the expense of intact forests, and another 28% came from disturbed forests (Gibbs et al. (2010)).

**Biodiversity loss** undermines functional diversity and can lead to persistent loss of ecosystem productivity. Steffen et al. (2015) suggest that we should aspire to achieve an no more than one extinction estimated per million species years (the background extinction rate), but that the planetary boundary is at about 10 extinctions per million species-years.\(^3\) Maxwell et al. (2016) recently analysed 8688 threatened or near-threatened species from the IUCN red list, concluding that agriculture and overexploitation are the most prevalent threats. Crop farming poses the single greatest threat, followed by logging. However, biodiversity loss is also an issue in marine environments, where it impairs the ocean’s capacity to provide food, maintain water quality, and recover from perturbations (Worm et al., 2006). Effects of human influence have been found in essentially all marine systems, and most areas are under strong pressure from multiple direc-

\(^2\)Using model simulation, Sunda and Cai (2012) show that the acidification from respiratory CO$_2$ inputs interacts in a complex fashion with that from increasing atmospheric CO$_2$ and that these pH effects can be more than additive in seawater at intermediate to higher temperatures.

\(^3\)By one extinctions per million species-years it is meant that if there are a million species on earth, one would go extinct every year, while if there was only one species it would go extinct in one million years.
Global fish production has increased 80 fold in volume since 1950, reaching around 144 million tonnes in 2006, and in order to accommodate standard estimates of population growth, an additional production of 75 million tonnes is needed (Rice and Garcia, 2011).

**Stratospheric ozone depletion** diminishes filtration of ultraviolet radiation from the sun, and creates increased risks to human health as well as to a great many other life-forms. These risks are mitigated as long as the ozone concentration remains above 275 Dobson Units (Steffen et al., 2015). Human activities contribute to ozone depletion primarily through the production of ozone-depleting substances, which are now well-controlled under the Montreal Protocol. Steffen et al. (2015) consider this boundary to be solved in the sense that humanity has taken effective action to stay within the boundary.

The remaining boundaries **Atmospheric aerosol loading** and **Novel entities** have not yet been quantified. Atmospheric aerosol loading, at high enough levels, decrease solar radiation at the surface, which can disrupt regional ocean-atmosphere circulation. Most significantly, perhaps, emissions of black carbon, sulfates, and nitrates, over the Indian subcontinent could eventually switch the monsoon system to a drier state. Steffen et al. (2015) use aerosol optical depth (AOD) as a single measure of the atmospheric concentration of these pollutants, and estimate that total anthropogenic AOD over the Indian subcontinent need to remain below 0.25 to avoid this possibility. They are however not able to quantify a global-level boundary for this process.

Novel entities can have unpredictable and disruptive effects on the Earth system. The invention of CFCs and the effect of their mass-production on the ozone layer provides an instructive example. Large-scale manufacture of chemicals that do not occur naturally, engineering of new organisms that can reproduce themselves, and mining of heavy metals, to name a few examples, may have as-yet undetected consequences for planetary stability. It is impossible to define an appropriate global planetary boundary here, the specific threat cannot be identified.

**In summary**, the key economic drivers of planetary pressures appear to be fossil fuel consumption and agricultural production, and to a lesser extent logging. Fossil fuel consumption is clearly a key driver of climate change and ocean acidification and via market forces has a myriad of other indirect impacts on the other ESPs. Agricultural production constitutes a relatively small portion of global economic output, but as we have seen, the share of physical resources used by this sector is not correspondingly small. Agriculture occupies nearly 40% of the Earth’s land surface, it contributes a substantial portion of our CO$_2$ and methane emissions, and accounts for over 90% of our use of freshwater, Phosphorous, and Nitrogen. Fossil fuel use and agriculture thus needs to be core components of any model that tries to capture how human activity puts pressure on the planetary boundaries. In addition, unlike other sectors, agriculture has long been the focus of policy because it provides an inelastic good, food. How this aspect affects policies regarding the planetary boundary is clearly of importance to understand. The aspects relevant for policy can therefore be explored by studying the markets for energy, land and food, which will be key markets in our model.
3 Policies for ESPs: An overview

A number of different policies to curb emissions of carbon dioxide (primarily from the burning of fossil fuels) have been discussed and implemented. Examples include carbon taxes, energy or output taxes, subsidies, quantity regulations (including the widely used cap-and-trade schemes), as well as a variety of voluntary programs or information measures (Bennear and Stavins, 2007; Lehmann, 2012). When discussing regulation covering a significant share of global emissions, the typically preferred option is some form of carbon pricing, either in the form of a carbon tax or in the form of an emission permit trading scheme. When all potential emitters face the same cost of emissions, emission reductions can be expected to be made in a cost-efficient way. Currently, 18 countries across the world – largely in the EU – have implemented carbon taxes while in total 35 are involved in emission trading (Kossoy et al., 2014).

For ESPs other than carbon, there are no overarching global policies in place, and different ESPs are regulated with differing objectives at regional or lower scales. What is noteworthy is the predominance of quantity-based policies in practice, allied often with administrative (‘command-and-control’) measures. Turning first to land-use, while there are many sectoral policies tackling different environmental problems at a local level, cross-country land use is typically not regulated in a regionally or globally integrated way (Wunder et al., 2013). A few exceptions in this connection are worth noting. One is the EU’s Common Agricultural Policy (CAP), a policy framework governing European farmers that has created a common market for agriculture in order to improve productivity ensuring a stable affordable food supply while guaranteeing living standards of farmers. It is the largest common policy in place within the EU, and has largely contributed to land use changes via the intensification of agricultural production in the EU (Wunder et al., 2013). Another is the policy of voluntary medium-term (10-15 year) land set asides in the U.S., largely for environmental reasons, involving a sizeable cultivable area (about 9.7 million hectares in 2017) with specific land-types and program goals (Hellerstein, 2006, 2017). Other examples of large-scale land-use policies includes one related to a voluntary international “regulation”, the UNFCCC’s REDD+ mechanism (adopted at the COP21 Paris meeting), primarily targeting climate change and focused on forests in certain regions of the developing world. An example of more direct and explicit land-use policy is one adopted by many of the countries experiencing rapid deforestation in Latin America, which regulate land-use through imposing requirements of maintaining forest reserves (Argentina, Paraguay) and other quantitative restrictions (de Waroux et al., 2016). A common feature of all land-related policies are ‘set-asides’, either mandated or voluntary.

Fertilizer-related policies on the other hand, have rarely been explicitly applied in practice, at least at large scales (Garnache et al., 2016), and those that have been applied are implicitly quantitative. The only region-wide, fertilizer policy in place is in the EU, via the Water Framework Directive specifying, for certain water bodies, maximum P- and N-fertilizer application rates (Amery and Schoumans, 2014; Oenema et al., 2011). The implied restriction is quantitative and location-specific. For the U.S., policies usually are of the pay-the-polluter, and target practices,
instead of outcomes, and can be viewed as a technology subsidy policy. Many reasons appear to have motivated the preference for quantity-based policies, including the often specific and localized nature of the persistent pollution problem that imply sizeable informational requirements for taxes. In addition, low price elasticity estimates (from limited studies) for fertilizer indicate that large price increases may be required to lead to sizeable reductions in application.

Regulation of (consumptive) water use is largely at regional/local scales, given the very local nature of water availability and transportation. While the details vary, these are largely quantitative restrictions or administrative measures. The literature on water use for agriculture is extensive, and what is striking is the very specific nature of these regulations, which differ in extent, scope and restrictions on irrigation water use, with different sources of water (surface and groundwater) also treated differently. In brief, surface water use is regulated at scales from river basin to the individual farmer (irrigator). Typically, agricultural requirements drive the large-scale determination of the ceiling of water use, although environmental and other requirements are increasingly also being prioritized (Grafton et al., 2011); subsequently, complex restrictions are imposed at varying levels to reflect the aggregate allocation. To illustrate only two important semi-arid settings, consider the Western U.S. and the Murray-Darling basin of Australia. Larger scale water use in both cases is determined by complex treaties and legal arrangements, with agricultural use largely driving the overall scale of water use. At the lower levels, there are a wide variety of water use regulations which vary widely in the two settings: in the Western U.S., farmer-level water use is determined based upon a complex series of water rights whose trade is limited and where sharing of deficit is complex (Libecap, 2010, 2014); while in Australia, similar rights are fully tradeable and deficit sharing is more transparent. In any case, not only are there no global policies, the specificity of settings of water use make it often difficult to arrive at generalized policies at a large scale, often even within country.  

Biodiversity received its first international address in the Convention on Biological Diversity, signed of by 178 nations including 100 heads of state in the United Nations’ Earth summit, Rio de Janeiro in 1992. This is a framework with little measurement and enforcement, no tangible resource commitment commensurate with the scale of the problem, and is generally considered to have failed to slow the problem (Tollefson and Gilbert, 2012). This is also the case for later international efforts with regards to biodiversity such as Pan-European Biological and Landscape Diversity Strategy (Delbaere, 2003).

In summary, the only global policy framework related to any ESP is a carbon-related set of policies, while regional policies to limit land-use (directly or indirectly), although infrequent, do exist; water, and nutrient policies – where they are in place – have thus far been local or regional, and driven entirely by agricultural policies and strategies, which indeed drive both use

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4Groundwater abstraction, an increasingly important source of irrigation in large semi-arid parts of the world, is largely unrelated, at least explicitly. Its overuse is contended to be one aspect of the looming global water crisis (Famiglietti, 2014; Rodell et al., 2009; Siebert et al., 2010). The few restrictions tend largely to be administrative, in the form of licensing and other administrative measures, with only in a few cases of quantity restrictions. Cost of extraction and transportation largely act as restraints on extraction (Shah, 2005).

5The only major exception to scattered localized policies, when it comes to the regulations of ESPs has been
flows of ESPs are used as inputs to production) and regulation.

4 An economic model of planetary boundary interactions

In order to study the economic interlinkages between the ESPs and the implications, in terms of change in pressure, of policies related to any one ESP upon the other planetary boundaries more systematically, we develop a general-equilibrium model of the economy with a focus on the key sectors identified in section 2.\(^6\) We will however, abstract from explicit consideration of the planetary boundaries stratospheric ozone, atmospheric loading and novel entities. The reason for not considering these explicitly are twofold. Firstly, stratospheric ozone is a process that Steffen et al. (2015) states as effectively being dealt with. Hence, additional policy considerations for this process does not appear to be warranted and we leave this out in order to not compromise unnecessarily on model transparency. Secondly, atmospheric loading and novel entities both have not been sufficiently quantified and thus have no global-level boundary associated with them. For aerosol loading this is mainly due to inconclusive scientific evidence at the global-scale and for novel entities it is simply unclear how this could ever be accomplished.

The sectors included are agriculture, biofuels, timber production, fertilizer production, phosphorus extraction, water utilities, manufacturing, energy services, fossil-fuel production, renewables and fisheries.\(^7\) Together with aggregate consumer demand the economic interaction between these sectors is determined within the model, by relative prices that equilibrate supply and demand. The resulting production and consumption decisions determine the allocation of resources in a market outcome and are also amenable to regulation through economic policies such as taxes or permits. Models of this type are commonly used to analyse the effect of policies which affect more than one activity or sector, and are ideally suited to analyse inter-linkages between different economic activities or sectors.\(^8\) They have also, as already alluded to, been widely used for climate policy analysis.

\(^6\)The Montreal Protocol, which was signed in 1987 as a first step in international efforts to protect stratospheric ozone. This agreement has led to substantial reduction ozone depleting substances, and to a gradual reversal of ozone depletion.

\(^7\)Our model is only loosely a general-equilibrium model since income is held fixed, and only the consumption bundle is endogenous. Given our focus on the short-run, however, the assumption of a fixed income seems not very restrictive.

\(^8\)The discussion in section 2 provides a strong basis for sectoral choice. The one sector that is perhaps not so clear is fisheries. One connection is via its link to the biosphere integrity boundary, as detailed in section 2. Another is the interaction with agriculture as a source of food whose demand is affected by changes in relative prices.

\(^9\)We note that many of the sectoral linkages considered follow the respective literature where possible e.g. competition for land is commonly considered in the biofuels literature (see e.g. Chakravorty et al. 2017 and references therein) and in studies which are focused on land use activities such as forest and biofuels (e.g. Steinbuks and Hertel 2016). Similarly, aspects related to sectoral structure such as fossil fuel extraction and fertilizer production or conversion of land, specification of production functions, are often similar in related modeling frameworks (e.g. Steinbuks and Hertel 2016 and Chakravorty et al. 2017).
4.1 Motivation

There has been a recent interest in how large-scale land-use change affects, and is affected by, climate change and climate policy. Interest in this aspect has been driven both by climate-related concerns as well as those related to food prices, particularly with the bio-fuel mandates and their land-related implications. Analyses thus far have consequently been: focused largely on attempting to account for the land-climate linkages, exploring the economic aspects involved and their implications for bio-physical changes; primarily numerical, typically in large computational models. Yet, there is, to our knowledge, no analysis of inter-linkages among economic activities and bio-physical processes that pertain explicitly to key ESPs most at risk of crossing (or already beyond) the planetary boundaries: fertilizer use, land-use, water, and ocean acidification, in addition to climate change. We are largely interested in highlighting the economic mechanisms that link the ESPs identified in Steffen et al. (2015), and explore the effects of key climate and land policies upon all the ESPs of interest, not just climate change or food supply. To this end, we emphasise the static economic linkages and key parametric assumptions involved, leading, we hope, to a better understanding of the overarching scope of policy options that arises from the planetary boundary framework of thinking. Our analysis thus complements, and builds upon, the more detailed analyses of specific questions related to land and biofuels (Bento et al., 2015; Chakravorty et al., 2017; Steinbuks and Hertel, 2016) and is intended to spur the development of richer models that take into account the multitude of dynamic interactions between economic and bio-physical systems.

The ESPs are, as already mentioned, driven by certain control variables (largely flow of chemicals or changes in stocks) and the planetary boundaries define the limits within which the control variables must lie to prevent irreversible or catastrophic damage to the earth system. We recall from Section 1 the control variables corresponding to the relevant ESPs, and note that these control variables are affected by specific economic activities detailed in Section 2: carbon (and carbonate ion) concentration; P, N flows into oceans; area of forested land; maximum consumptive water use; measure of change in biodiversity. The control variables relating to each of the ESPs are affected by the specific economic activities detailed in Section 2. The two economic aspects we are most interested in understanding, using the conceptual framework outlined above, are: substitution between different inputs (e.g. land and non-land inputs in agriculture) and demand elasticity for key goods (e.g. food). These are the primary channels through which policies focused on climate change can have substantial implications for other ESPs (and vice-versa). We provide first a textual description of the important elements of the model and which ESPs relate to which economic activities, including brief motivations for choices, before proceeding to a formal, mathematical description.
4.2 General model outline

We begin first with land-use change, focusing on global-scale aspects of land use that are of particular interest. Land can be used for agriculture, timber production or be conserved for other purposes such as recreation, ecosystem service generation, biodiversity preservation and carbon sequestration. The services provided by land conservation thus trades off the net carbon emissions and biodiversity loss connected to the expansion of agriculture and forestry. Food and biofuel are derived from the same composite agricultural good, meaning that they compete directly with one another for all inputs including land. Land is a primary input to agriculture and is somewhat substitutable with other intermediaries such as water, fertilizers or energy services, capturing the potential intensification via other inputs, reflecting agricultural land dynamics in reality. We consider a single nutrient source (fertilizer) in our model, produced using phosphorous mineral and fossil fuels. We thus do not model nitrogen-based fertilizers explicitly and instead assume it is directly linked to fossil fuel use, as a primary production factor. While clearly an abstraction, this greatly simplifies our model without any substantial loss of insight at the global scale. The quantity of nutrients fluxed to freshwater clearly is a function of application and of use efficiency, the latter of which has been pointed out as being a key parameter in prior literature.

There are seven intermediate goods in the model: nutrients, composite energy, phosphate extraction, freshwater and energy extraction/production (fossil, biofuel, renewables). The energy-services good simply combines all sources of energy (except the fossil fuels used as an input in nutrient production) – fossil fuels, bio fuels and renewables – into a composite, and is used in agriculture and manufacturing. Fossil energy production uses fossil resources as the sole input. The supplies of fossil fuel, renewables, phosphate, freshwater and fish, are simply represented by (convex) cost functions. Further, we have four consumption goods: food, land, timber and a composite good are the inputs to utility. Food is an aggregate of agricultural goods with fishery. As previously mentioned, the land use that enters directly into the utility function can be interpreted as used for e.g. recreational or ecosystem-service purposes. Note that, in view of the short-run or equilibrium interpretation of our model, we treat both labour and capital as fixed inputs. While this is standard for capital, the fixed labour assumption, while clearly a simplification, can be justified both by the resulting simplification and the focus of our analysis purely on input substitution. We follow the modeling literature (e.g. Bento et al. 2015; Chakravorty and Hubert 2012; Chakravorty et al. 2017; Steinbuks and Hertel 2016) in not accounting for the negative effects of different flows of ESPs (e.g. nutrients, climate change) upon welfare; consequently, all policies are exogenously chosen. That said, clearly an accounting of at least the direction of change in the ESPs is important and section 4.4 details its implementation in our model. Here, we briefly detail

\footnote{In any case, in the absence of specialised labour in the model, this is clearly a more sensible choice than assuming freely mobile labour across disparate sectors or ad-hoc specifications of labour mobility across sectors. Finally, we note that low estimates of the (short-run) elasticity of substitution between labour and other inputs in agriculture suggests that this assumption is tenable as a first approximation.}
how carbon flows are accounted for, since they contextualise some of our production structures. Carbon flows in our model are simple, with no intention of a comprehensive accounting for all flows. Carbon accretion results from deforestation and from the burning of fossil fuel either for providing general energy services or for producing fertilizers. For simplicity, fossil fuels (and phosphorous, freshwater, renewable fuels and fish) are produced at a given exogenous cost that increases in produced quantity without explicitly specifying the used inputs, hence are not associated with any carbon emissions. All sectors are assumed competitive, with each producer treating input and output prices as given. When describing the model, we refer to the climate policy as a carbon tax. The analysis does, however, apply equally to other ways of pricing emissions such as a permit trading scheme. Carbon taxes are imposed on production of fossil fuels, rather than on their consumption (i.e. consistent with application in the real world e.g. the EU ETS). Hence, the effect of a carbon tax upon consumption of final goods, or upon utility, is via altered relative prices for the intermediate sectors.

4.3 Model details

We describe here the key components of our model framework. The model will be investigated as a decentralized equilibrium and we describe the different sectors in terms of the profit-maximization problem facing the representative producer in the sector. We note that key aspects of the modeling framework are representative of current economic reality, and are drawn from existing model set-ups, narratives and parameters, where they exist (e.g. Chakravorty et al. 2017; Steinbuks and Hertel 2016). Noteworthy model aspects include: imperfect substitutability between the three sources of energy, fossil, renewable, and bio-energy; competition for land between food and bio-fuel; and the dependence of fertilizer production upon fossil fuel usage.

Land is homogeneous in quality, with endowment $L$, and has three uses: as managed lands (for agriculture), $L_A$, land used for commercial forestry to produce timber, $L_T$, and natural un-disturbed/un-used lands, $L_U$ (which provides utility directly, e.g., via recreation), summing to the endowment,

$$L = L_A + L_T + L_U.$$  

Using land for agriculture or timber production entails a convex clearing cost $c_A(L_A), c_T(L_T)$ respectively. These costs may be considered as consisting of a per-hectare cost, potentially increasing in the amount of cleared land.\footnote{Note that these costs are intended to be linear in the clearing, and thus, one may anticipate that the total costs are of the form $c_A(L_A)(L_A - L_A)$, with $L_A$ the baseline land used for agriculture (instead of $c_A(L_A)L_A$ in eq.(2)). For simplicity, we do not use this formulation but note that the cost function may be numerically adjusted to reflect this aspect. In any case, for many reasons, this aspect will turn out not to matter at all for our computations.} The agricultural sector, which is an intermediate sector, produces a homogeneous good, $A$, via a production function, $A(.)$, and the representative farmer solves the problem

$$\max p_A A(L_A, P, W, E_A) - p_L c_A(L_A)L_A - p_P P - p_W W - p_E E_A.$$  

12
where $P$ is fertilizer use, $W$ is water use and $E$ represents energy services. The prices are $p_A$ for the agricultural output, $p_L$ for land (not including the clearing cost), $p_F$ for fertilizers, $p_W$ for water and $p_E$ for energy services. The Agricultural good can be used for either food production, thought of here as cereals and animal products, $A_F$, or biofuels $A_B$:

$$A = A_F + A_B.$$  \hfill (3)

The timber sector produces timber with land as the sole input and the associated firm/sectoral maximization problem is

$$\max_{L_T} p_T T(L_T) - p_L c_T(L_T) L_T.$$  \hfill (4)

The fertilizer intermediate is produced through a process in which fossil fuel $E$ and phosphate $P$ are combined into usable fertilizer $P$ (the output may be thought of as representing a combination of nitrogen and phosphorus fertilizer, where fossil fuel is used to capture nitrogen), with the representative firm’s maximization problem being

$$\max p_P (E_P, P) - p_E E_P - p_P P.$$  \hfill (5)

The following sectors have a (gross) cost function that is increasing and convex\textsuperscript{11} in output or extraction, and are detailed next: phosphate, fish, water, fossil fuels, and renewable energy.\textsuperscript{12}

The phosphate industry extracts phosphate $P$ based on an extraction-cost function $g_P$, leading to the representative firm’s maximization problem being

$$\max_P p_P P - g_P (P).$$  \hfill (6)

The representative firm in the fishery sector simply “produces” fish, $F$, at an increasing and convex cost $g_F$, with maximization problem

$$\max_F p_F F - g_F (F).$$  \hfill (7)

The representative firm in the water sector extracts water under a cost function $g_W$, with maximization problem

$$\max_W p_W W - g_W (W).$$  \hfill (8)

The representative firm in the fossil sector extracts fossil energy, $E$, with a cost function $g_E$.

\textsuperscript{11}Costs are gross in the sense that total, not per unit, extraction (production) cost is increasing in extraction i.e. $C(.) \equiv g(.)$.

\textsuperscript{12}We do not make specific distinctions between different transformations (e.g. fossil fuel to refined product) and between different combinations (e.g. equivalent quantities of biofuel and refined fossil fuel products). While straightforward with the application of relevant coefficients, it is not pursued here since our manner of solving the model does not require these details (see section 4.5 and 4.6). Consequently, output of biofuel and fossil fuel sectors can be either directly used or combined without the use of any conversion factors.
We assume that the extraction firms pay the fossil-fuel tax, solving the problem

\[ \max \frac{p_E}{1 + \tau_E} E - g_E(E), \]  

(9)

where \( \tau_E \) is the tax on fossil-fuel use. The extracted fossil fuel is divided between the fertilizer sector and the energy-services sector

\[ E = E_F + E_P. \]  

(10)

The representative firm in the renewable sector produces (renewable) energy \( R \) and solves maximization problem

\[ \max p_R R - g_R(R). \]  

(11)

We note that the three sources of energy, fossil fuels, bio-fuels, and renewables need not always be used in every sector. Indeed, the fertilizer sector uses fossil fuels directly while production of the composite good \( Y \) (see below) and agriculture uses energy services provided by an intermediate energy sector that combines renewables, fossil fuels and bio-fuels into energy services \( E \), solving:

\[ \max p_E E(A_B, E_F, R) - p_A A_B - p_E E_F - p_R R. \]  

(12)

The three sources of energy are imperfect substitutes, meaning that the price of the composite energy good will vary with composition. We will use a single CES function for energy services in our simulations, implying identical elasticity of substitution between the three energy sources.

The produced energy services are divided between composite production and agriculture

\[ E = E_A + E_Y. \]  

(13)

Finally, the composite good \( Y \) is produced, as already indicated, using energy services

\[ \max p_Y Y (E_Y) - p_E E_Y. \]  

(14)

Denoting by \( I \) the aggregate income levels, the representative household solves the following utility-maximization problem

\[
\max_{A_F, F, Y, L_U, T} U \left( F(A_F, F), \tilde{F}(Y, L_U, T) \right) \]

\[ \text{s.t. } p_A A_F + p_Y Y + p_F F + p_L L_U + p_T T \leq I. \]  

(15)

The structure of utility is somewhat involved, and is worth mentioning. Utility is derived from either consumption of a composite (of agricultural and fishery products) good called ‘food’, \( F \) and another composite (of the manufactured good, timber and recreational from undisturbed land), \( \tilde{F} \). We assume that, food and other commodities are poor substitutes in utility, while the
inputs into these composites are generally better substitutes. That is, agricultural goods and fishery products are considered to be moderately substitutable in the composite food commodity; similarly, the commodity represented by the non-food nest is also assumed to have a fair amount of substitutability among the constituent goods. For our simulations, in fact, we assume, in common with much of the modeling literature, that the utility function and the two nests are of the CES variety.\footnote{We note that the use of two nests ensures that the desired substitutability patterns are obtained with the simpler CES functions. CES functions also ensure that the consequences of our assumptions are transparently visible. Finally, the fixed nature of income and the moderate changes induced by our policies ensure that the flexibility offered by more complex functional forms is not of much consequence for our simulations.}

The production structure of the agricultural sector, identified as key from our perspective, is also modeled as a nested CES in our simulations. The upper level nest is between land and other inputs which, following prior literature, is parameterized as being rather substitutable. The non-land nest however consists of inputs that are often modeled as having little substitutability. Motivated by this, we assume there is little substitutability between the primary non-land inputs of water, nutrients and energy (as is often assumed in many disaggregated CGE models such as the GTAP (Aguiar et al., 2016). As we detail subsequently, we will not need to put functional forms for all functions involved in the model; wherever necessary, however, we choose the CES form of the production function.

4.4 Accounting for policy-induced changes in ESPs

Consistent with the spirit of our approach, which is a focus on understanding the underlying market linkages, we will only consider changes in the fluxes of the ESPs consequent to the sectors and linkages most relevant, eschewing a complete accounting of all possible fluxes for every ESP.\footnote{Nonetheless, since our model accounts for the sources of almost all of the important fluxes to different ESPs, it is possible to extend our analysis by explicitly computing the magnitudes of different fluxes resulting from a given policy. This is a task we leave for the future.} This, we emphasise, is not a drawback of our model since we do not, in any case, explicitly model the externality that is imposed on the economy by the interacting ESPs. Thus, the relevant policy (e.g. a carbon tax) chosen will be exogenously imposed from the perspective of this model. We note that these are commonly used aspects in many economic models that focus on specific sectors of the economy (e.g. Bento et al. 2015; Chakravorty et al. 2017; Steinbuks and Hertel 2016). Consequently, we take the approach of indicating the direction of change, to a first approximation, to the ESPs of interest resulting from imposing a given policy. In light of the challenging nature of specifying a form for the “damage function” for even a single externality, the climate (Weitzman, 2010), this approach yields the substantial benefit of not needing to parameterise the relevant form of “damages” resulting from each and every externality. In any case, our interest is centered around understanding how a policy aimed at one ESP will affect other ESPs, and the implications of the movement towards (or away from) the relevant boundary: the magnitude of these effects are left for future research. A key step in attaining the required understanding is a specification of the relationships between the variables in the model and the
control variables related to different ESPs. This step is taken up here.

Some of the boundaries are directly related to model variables. These boundaries are land use ($L_A$, $L_T$ and $L_U$), water use ($W$) and fertilizer use ($P$), since these are precisely the fluxes (and in turn the control variables) which determine whether the relevant ESP is moving towards the boundary or away from it. The other boundaries depend in a more complicated way on the model variables. More specifically, climate change is exacerbated by conversion of land from unused to other uses (especially agriculture) and by use of fossil fuels. We can thus write a climate indicator as a function

$$C(E, L_A, L_T),$$

with the understanding that increases in agricultural land use (Searchinger et al. (2008)) and fossil fuel use leads to increased carbon emissions. Ocean acidification is more or less directly related to climate change, and the climate indicator functions also as an indicator for the direction of change for this ESP (with the caveat that the magnitude will be clearly influenced by feedbacks etc. that clearly cannot be considered in a static framework such as ours).

Biodiversity is the most complicated of the boundaries since it is affected by many different factors. In terms of our model variables, biodiversity is worsened by climate change, increased water use, increased fertilizer use, fishing and conversion of unused land (as detailed in section 2). We could thus write a biodiversity indicator as


To summarise, the direction of change in these indicators, if easily determined, should suffice to determine, broadly, the direction of the effects of any policy (for one or many ESPs) upon each ESP.

### 4.5 Solution Approach

There are essentially two approaches available now to use the model developed to help answer questions of interest. We detail next the main ideas involved in our preferred approach. We note that, as already detailed above, there are no explicit externalities in our model, and policies are applied exogenously; and second, all sectors are assumed to be competitive and the market always clears. Thus, we can work with the de-centralized equilibrium which, as always, may be obtained by examining the FOCs (at least in principle), and determine the optimal outcomes therefrom. There are about 30 unknown objects of interest in the model, a combination of quantities and prices, which may be solved for using the equilibrium conditions that result from the FOCs as well as a few constraints. After eliminating prices, there are, in total, 19 resulting equilibrium conditions that may be expressed in terms of the 19 quantities. Being exogenous, policies represent parameters that are known in advance; denote a generic “policy” pertaining to any one ESP by $\tau$, noting that policies that appear as constraints will in fact appear separately, when the constraints are applied subsequently (again, in principle). Let $Q_i$ denote the generic
\(i^{th}\) quantity, leading to the \(j^{th}\) equilibrium condition being
\[
G_j(Q_1, \ldots, Q_{19}; \tau) = 0. \tag{18}
\]
The key point to grasp is that all resulting equilibrium quantities are functions of the policy i.e.
\(Q_i = Q_i(\tau)\) (with abuse of notation).

There are now two ways to proceed: the first way is by simply solving the set of resulting non-linear equations (thereby obtaining all the quantities, and therefore, prices); the second is by tracing out only marginal changes w.r.t. the policy, \(\tau\) upon equilibrium quantities (and/or prices). The latter approach may be easily understood by considering the total derivative w.r.t. the policy, and leads to the following system of equations, with the \(j^{th}\) equation being
\[
\sum_{i=1}^{19} \left[ \frac{\partial G_j}{\partial Q_i} \frac{dQ_i}{d\tau} \right] + \frac{\partial G_j}{\partial \tau} = 0. \tag{19}
\]
This is a system of 19 equations in 19 unknowns, the \(\frac{dQ_i}{d\tau}\), and is most useful because of the linearity in the unknowns. The benefits of this second approach include the far fewer number of parameters (and functional forms) needed to solve the system, as well as an escape from numerical issues encountered with non-linear systems.

We note that in view of the fact that we do not need to parameterise the functional form for damages related to each ESP, the lack of quantity information, \(Q_i\), is not as limiting as might appear at a first glance. To understand this, note that to learn the direction of movement of each ESP, we need, except for the climate change and bio-diversity ESPs, only information regarding the change in the fluxes of the respective ESPs. As regards climate change, we need ideally both the direction and magnitude of each of the two changes (ignoring timberland, for the present): change in land use (emissions resulting from which are often termed indirect) and fossil fuels (related emissions are termed direct). Clearly, if both change in the same direction, knowing the direction is sufficient, while the magnitude information is required only when they change in opposing directions.\(^{15}\) Finally, for bio-diversity, provided the major drivers (land use, primarily) move in the same direction, it is possible to see the overall direction of change (movement towards or away from the boundaries) resulting from a policy change. If, however, the major drivers move in opposite directions, then it is difficult to put a sign on the change. We note, however, that this is a problem less to do with lack of a quantity information computed in our model and more to do with the well documented difficulties in quantifying the role of different drivers of large-scale biodiversity changes.

\(^{15}\)In more detail, direct and indirect emission changes resulting from e.g. a carbon tax may be computed as follows to enable a comparison: percentage change in fossil fuel usage resulting from a tax can be converted to a percentage change in carbon emissions (via a conversion factors that converts each $ value added to CO\textsubscript{2} equivalent emissions). Similarly, land use related changes (e.g. a 1% increase) can be converted to CO\textsubscript{2}-equivalents.
4.6 Parameterisation and functional forms

From the discussion of our solution approach above, we recall that the object of interest in our case is the change in the endogenous variables induced by the change in the tax, rather than the actual quantities, \( Q_i(\tau_E) \), where \( \tau_E \) is an exogenous fossil fuel tax. More precisely we will solve for relative changes in the quantities, denoted by a "hat" above the variable:\(^\text{16}\)

\[
\hat{Q}_i = \frac{1}{Q_i} \frac{dQ_i}{d\tau_E}.
\]  

(20)

This, as we will now detail, significantly reduces the parameters required to be collected. To see the intuition, note first that the system of equations implied by eq. (19) means that the numerical values required to derive the effects of a policy change are the “coefficients”, \( \partial G_j / \partial Q_i \). This is potentially a significantly smaller set of values and it is, in fact, possible to break these down into components (e.g., elasticities and cost shares) that have been previously estimated, particularly for CES production and utility functions. Define three distinct entities, all related to elasticities, and consider for concreteness the case of nutrient production function, denoted \( P \), with inputs phosphorous \( \mathcal{P} \) and fossil energy, \( E_P \): \( \Gamma_{E_P}^P \equiv \frac{E_P}{\mathcal{P}} \frac{\partial \mathcal{P}}{\partial E_P} \), \( \Gamma_{E_P,P}^P \equiv \frac{\partial^2 E_P}{\partial E \partial \mathcal{P}} \mathcal{P} \) and \( \Lambda_E \equiv \frac{\partial \mathcal{P}}{\partial E} E \). These are, respectively the own and cross-price elasticities, and the (inverse of the) “supply function” for fossil fuel (similarly for phosphorous production). Now consider applying equation (19) on the FOC for nutrients (see Appendix B for fuller details), when relevant production and utility functions are assumed CES (see Appendix B.2). The resulting comparative statics condition can be written as:

\[
\sum_{X \in \{E,\mathcal{P}\}} (\Gamma_{E_P,X}^P - \Gamma_{E_P,X}^E) \hat{X} = \frac{1}{1 + \tau_E} \hat{E} - \Lambda_P \hat{P}.
\]  

(21)

To the extent that estimates of the three entities appearing in eq. (21) – \( \Gamma_{E_P,X}^E, \Lambda_E, \Lambda_P \) – can be obtained, computation can focus on estimates of the hatted objects which, as detailed above, have the interpretation of proportional changes.\(^\text{17}\) We note that two types of information are required to compute these entities, CES production and utility function parameters, and cost share and cost function parameters. We turn next to providing the details of both aspects.

Broadly speaking, we parameterize the model based partly on data extracted directly from the widely-used GTAP data base and partly on estimates of substitution, demand and cost estimates gathered from various sources in the literature. Since we are only engaging in comparative statics and never actually solve for the equilibrium quantities we require only a subset of the estimates.

\(^\text{16}\)These can be interpreted as a linear approximation of the percentage change in the variable induced by a one percentage point increase in the fossil fuel tax. Assume, for instance, that we get \( Q_i \approx 2 \) and consider a one percentage point increase in the tax rate, \( \Delta \tau_E = 0.01 \). We would then get \( \frac{1}{Q_i} \Delta Q_i \approx \hat{Q}_i \Delta \tau_E = 0.02 \). Hence, a one percentage point increase in the tax induces a two percent increase in the quantity. Note an alternative, and straight forward, interpretation of hatted variables as proportional changes, consistent with the standard interpretation in growth models (where they are termed growth rates).

\(^\text{17}\)The model can also be solved for (changes in) prices, using an approach similar to that for the hatted quantities. We solve for, and discuss, both.
that would otherwise have been needed. This includes four types of quantities, elasticities of substitution, price elasticities of supply, quantity shares and various expenditure or factor shares. In total this still implies roughly 31 parameter estimates which need to be acquired. In appendix B.5, we have included tables with parameter values and their sources.

5 Policies and Effects on boundaries

We consider two policies, a carbon tax and a land-use restriction (e.g. disallowing conversion of land from the baseline), for reasons detailed in Section 3. Two major outcomes are of particular interest to our analysis: the effects of a carbon tax on indirectly-related boundaries e.g. nutrients and land use; and the effects on food prices. Given that we consider a policy directly involving land-use, an investigation of food price increases allows our work to also be placed in the context of related literature where effects on food prices are at the forefront (e.g. Chakravorty et al. 2017; Rosegrant et al. 2008b).

It is also of interest to explore the precise channels of substitution that turn out to be particularly important, as well as the role played by the degree of substitutability between food and non-food consumption. These aspects, we believe, help us link welfare related questions most commonly addressed by economists with the effects on PBs, which are the focus of analysis in the science and policy literature (exemplified by the ad-hoc prescription of halving carbon emissions every decade here from, in Rockström et al. 2017). We will discuss, for each policy, and policy combination, the effects of (marginal) changes in the policy on features of interest to us e.g. prices, quantities, and fluxes of ESPs (such as the carbon flux). While we refer to changes induced by policies as being “direct” or “indirect” as an aid to intuition, we note that in the model all changes are simultaneous (i.e. markets clear simultaneously) and the magnitudes of indirect changes may be sizeable.

5.1 A global carbon tax

The direct effect of imposing a tax on fossil-fuel use is to increase the (after tax) price of fossil fuel. This, in turn, exerts a direct effect on fossil-fuel use and indirect effects on all endogenously determined variables which use fossil fuel as an input to production (or consumption). The general-equilibrium effects on all endogenously determined quantities and prices are shown graphically in Figures 1 and 2 respectively. The bars in the Figures are interpreted as the percentage change in the quantities and prices (i.e. the hatted variables) in response to a one percentage point increase in the tax on fossil fuel. Our model results are best understood by bearing in mind two aspects (see brief discussion in section 4 above): the elasticity of substitution for fossil fuel, and the elasticity of demand for a good in final consumption. We organise our discussion around key points of interest and discuss both quantity and price responses, noting that the former is of most interest when the effects upon ESPs are considered while the latter is of considerable importance for assessing and understanding welfare implications.
Turning first to the quantity response summarised in figure 1, we see that a percentage point increase in the carbon tax increases the fossil fuel price by approximately 0.79 percent and reduces total fossil-fuel use by approximately 0.62 percent. This increase in price leads both to an overall increase in production costs for fossil-fuel-reliant sectors (thereby reducing their output) and to a change in the factor input composition (due to changes in relative prices). For example, in the energy services sector, the increased relative price of fossil fuels causes a substantial increase in the demand for both renewables (0.40%) and biofuels (0.61%).¹⁸ The total reduction in fossil fuel use comes mostly from a reduction in demand as an input in the production of energy services (that decreases by 0.63%) and to a much lesser extent from a decrease in fossil fuel used for fertilizer production (0.0013%). The difference in quantity responses for the different uses of fossil fuel comes largely from the difference in substitutability between fossil fuel and the other inputs in the different uses, and in substitution between fossil and other fuels. In fertilizer production, there is a very low degree of substitutability between fossil fuel and phosphate (and no possibility of other fuels being substituted for fossil, in the short-run) while there is fairly good substitutability between different energy sources in the provision of energy services. From Figure 2 we can see that the fossil-fuel price increases by about 0.79 percent.¹⁹ The prices of energy services increases by 0.76 percent (consequent to the total reduction in energy services) while the price of fertilizers increases by 0.026 percent. An important factor behind this differing price response is the sizable difference in cost shares of fossil fuel in these two sectors, with the share in energy-services production being far larger (see calibration in section B.5).

The net effect of this tax upon land use is determined by a combination of opposing effects. Turning first to agricultural demand, land use in the agricultural sector increases due to the increased relative price of non-land inputs (energy and fertilizer). At the same time, the production of the manufactured composite decreases due to the increase in the fossil fuel price. This increases the marginal value of consuming other non-food goods and hence the demand for land in other uses (for timber and recreation). This increase in the demand for land for non-agricultural use counters the increase in demand from agriculture. Together with land conversion costs, this results in a relatively moderate change in land use. Overall, there is a small increase in land use in the agricultural sector (at less than 0.3% and a decrease in both other uses.

Turning to the change in the demand for water, there are again a number of competing effects. The change in the use of other inputs has an ambiguous effect on the marginal product of water in agricultural production. Firstly, the decreased used of other non-land inputs (fertilizers and energy services) decreases the marginal product of water in the non-land composite input. Secondly, the decrease in the non-land composite and the increased land use increases the marginal product of the non-land composite in agricultural production. Hence, the net effect on the marginal product of water in agricultural production is ambiguous and will depend on

¹⁸Note that, being proportional, the percentage changes in renewables and biofuels need not add up to the change in energy services, not even to the direction of change (which, it will be seen, is negative).
¹⁹It is worth noting that imperfect substitutability between renewables and fossil fuels (as well as between renewables and biofuels) ensures that the price of renewables and bio-fuels increases, leading to the above-noted increase in price of energy services.
substitutability in the different levels of the agricultural production function. In addition to that, the increased price of agricultural output tends to increase the value of water use. The net effect on the marginal value of water use, for the chosen parameterisation, is a very small reduction (about 0.0003%).

Turning to an analysis of price changes, it is seen that increased agricultural production costs affect both the price of, and demand for, food. First, there is a direct increase in the price of agricultural products (of about 4%). However, since fish and food from agriculture are (weak) substitutes in utility, an increase in the price of the latter is not fully translated into higher food prices. Instead, substitution possibilities between fish and agricultural products partially dampens an otherwise larger increase in the price of food.\textsuperscript{20} As can be seen in Figure 1 this results in a reduced demand for food from agriculture, an increase in the demand for fish, and an increase in the price of food by only a small amount (0.03%).\textsuperscript{21}

In conclusion, we find that an increase in the carbon tax of 1% leads to increase in price of not

\textsuperscript{20}More explicitly, in the absence of fishery, with agriculture being the only source of food, agricultural price increases would be slightly larger than observed here.

\textsuperscript{21}As to the prices of goods for which a specified extraction-cost curve implies a supply curve, prices and quantities will move in the same direction (i.e. a quantity increase will always imply a price increase and vice versa). This holds for water, fish, renewables and phosphates. For fossil fuel this holds for the pre-tax price but the reported price is the price including tax and the reported effect is the net of the increase in the tax rate and the decrease in the pre-tax price (induced by the decreased demand due to the increase in the after tax price).
just fossil fuels but all goods which use fossil fuel for production. In addition, due to imperfect substitutability, energy services production falls, despite sizeable increases in both renewables and biofuels. Finally, food production falls and price increases, bio-fuel production increases, and there is a slight increase in land use for agriculture.

5.2 Introducing a land-use policy

We now consider the introduction of a land policy, which we view as a complementary policy to prevent (minimise) “leakage” from the energy-related climate policy. In essence, we view this set of policies, a carbon tax supplemented by a land-use restriction on agriculture, as a second best set of policies. Motivated by this approach, we pursue a land-use policy restricting agricultural land-use to the baseline levels. In light of the potential for intensification, this policy, on aggregate, may not be as extreme as it can otherwise seem (at least, in the context of our model). In any case, we implement this policy as disallowing increases in land use for agriculture beyond the baseline. Increases in timber land are however allowed, since they are likely beneficial from a climate-change-mitigation perspective.

\[22\]

Clearly, the first best policy, abstracting away from many other considerations, would involve setting a carbon price that accounts for the change in land use and any climate-relevant effects and changes economy-wide. This is however not how many models often used in policy compute the carbon price and therefore is less relevant from a practical policy perspective.

\[22\]
Figure 3 illustrates the changes in responses of quantities to this additional policy. In the graph, the bars correspond to the difference between the response to the carbon tax with and without the imposed restriction on land use in agriculture. An obvious difference is that there now is no increase in agricultural land use (the negative bar represents the difference between the increase without the land restriction and the zero effect of the tax with the land restriction). In fact, given the way timber and recreational land enters the utility function (as separate arguments), and the specification of the timber production function (with land being the only input), fixing agricultural land at the baseline value will imply that both other land uses will be unaffected – from the production side – by the carbon tax as well. Furthermore, the price of timber will be proportional to the price of land implying that the percentage change in both prices will be the same.

The effect of the tax on fossil fuel use is almost identical, illustrating that the feedback between land-use and fossil fuel is very limited (largely via changes in agricultural inputs). In the agricultural sector, the reduction in water use is somewhat larger while the reduction in energy use is somewhat smaller. The difference in the effect on fertilizer use is extremely small, consistent with the assumption that primary agricultural inputs (water, fertilizers and fossil fuels) are poor substitutes. The restriction on land-use thus implies that the main channel for mitigating the increase in production costs resulting from the carbon tax is via reduced output. This effect is evident from the reduction in total agricultural production and both its products – biofuels and food. To summarize, the land-use restriction removes compensation by the extensive margin as a result of the increase in input prices induced by a carbon tax; thus, the resulting adjustment is via output reduction. More importantly, the reduction in food production is slightly larger than that in biofuels. The reduction in food production is to some extent compensated for by increased fishing. These changes are also reflected in the changes in prices shown in Figure 4. The larger negative effect on food production is reflected in the larger increase in the food price. Furthermore, there are larger price increases for agricultural goods, fertilizers, fish and water.

To summarise, restrictions on increased land usage in agriculture has the anticipated effect of increased food prices; what is interesting is that despite this, bio-fuels are prioritised. While the usage of non-land inputs does in fact increase, it is not sufficient to compensate for the restrictions implied by energy taxes and low substitutability, leading to the observed reduction in all agricultural production activities. Finally, since the land use restriction leads to a larger effect of the carbon tax on agricultural production and food prices, it will also lead to a larger increase in the price and production of fish.

5.3 Implications for ESPs

The effects of a carbon tax can be quickly summarised as leading to: reduced fossil fuel usage, fertiliser usage and production of food (leading to price increases); increased agricultural land use (reduced land use for timber); increases in renewable, bio-fuel production; reduced water use although by very small amount. Taken together, it is clear that most ESPs move towards
the interior of the safe zone. For the carbon system (i.e. the climate and ocean acidification boundaries) the effect is actually ambiguous due to the increased emissions from land use. Given the sizeably larger carbon intensity of fossil fuels relative to land use, the carbon ESP should also be moving in the right direction, with the only difference being that the quantum of movement is not by the full amount implied by the carbon tax due to the “leakage” via increased agricultural land use. Finally, the implications for biological integrity is somewhat ambiguous due to differing directions of change in key drivers: land use and fishery increasing but water use, fertilizer and carbon fluxes reducing, see equation (17).

When the carbon tax is supplemented with a land-use policy, disallowing any additional land conversion for agriculture, the resulting changes to the ESPs are, by and large, small. In particular, the direction of all changes are unaffected including, for reasons already discussed, that of land use for timber and recreation. Hence, the carbon ESP unambiguously moves towards the “safe zone” while the land-use ESP’s control variable shows no movement. There is also a small reduction in the pressure on the water-use ESP, and virtually no change in the pressure upon (flux from) the nutrient ESP. Compared to the case without a land-use policy, there is less pressure on the biodiversity boundary from climate change, land use and nutrients but more pressure from fisheries. Consequently, it is not obvious that the pressures upon biological

Figure 3: Percent difference in the level response between the land and no-land policy from a one percentage point increase in the carbon tax when land use is not allowed to increase in agriculture.
Figure 4: Percent difference in the price response between the land and no-land policy from a one percentage point increase in the carbon tax when land use is not allowed to increase in agriculture.

integrity are unambiguously reduced with an (additional) land-use policy.

In conclusion, therefore, it is seen that the combination of a carbon tax and a land use policy appears able to decrease almost all of the planetary pressures. The increased exploitation of fisheries that result from higher food prices, however, have negative consequences for the biodiversity boundary. Additional fishery-specific policies could be formulated to minimise or manage this aspect. It seems unlikely that this would change the signs of the combined set of policies on the other boundaries. It would, however, further limit food production or lead to food price increases.

6 Conclusions and Discussion

This paper has developed a general-equilibrium model that integrates key markets related to the ESPs considered in the PB literature: fuel, food, land, nutrients, water and fishery. This framework was used to explore two key aspects: the effects of a policy intended to account for a subset of the ESPs upon others; and the minimal set of most plausible policies to help move as many of the ESPs as possible in the right direction. The policies chosen were the carbon tax, targeting the climate change and ocean acidification ESP, and a land-use restriction
in agriculture, targeting the land-use change and climate ESP. Using aggregate global data and parameters drawn from the literature, we explore the direction of movement of the ESPs consequent to the considered policies, individually and in combination.

We highlight two key findings. First, despite as many as six ESPs that are considered separately, a combination of two policies – the primary policy of a carbon tax and a supplemental policy of regulation of agricultural land use – is sufficient to ensure that most of the planetary pressures are reduced. This, rather surprising, finding stems from the nature of sectors exerting the pressure. With only two sectors, fossil fuels and agriculture, largely responsible for the bulk of the pressure the policy problem turns out to be manageable once these linkages are carefully examined. Consequently, one policy for each of the sectors is largely, although not completely, successful in reducing all planetary pressures.

Second, while planetary pressure is reduced, these two policies together need not be welfare enhancing. This is because the policy view considered here is partial, in that welfare has not explicitly been taken into account. Thus, without making an explicit trade-off between broader welfare, including food prices, and progress towards easing planetary pressures, it is difficult to speak of the policy mix chosen here as being optimal. There may also be a time-dimension involved, since the negative effects of many of the planetary pressures are likely realised far in the future while the costs, in terms of food prices, are immediately apparent. Nonetheless, given the smaller-than-expected magnitude of (marginal) price increases resulting, our results in fact are indicative of potentially lower welfare costs of carbon tax-like policies. It is also comforting that, at least indicatively, these are confirmed by recent studies of carbon policies.

Our study also highlights the importance of the market-related linkages between the six ESPs under consideration: it is the strength of these linkages which is largely responsible for ensuring that two policies are sufficient to reduce the pressures on all-but-one ESP. Put another way, while the ESPs differ in many important ways and in timescales and spatial scales, they are strongly related in economic terms, being largely driven by the agricultural sector and/or dependent upon fossil fuels. Our study also relates to the literature in economics evaluating the effect of addressing multiple externalities with a single (or a few) instruments as well as the previously mentioned theoretical literature on using multiple policies to address a single externality (Bennear and Stavins (2007)). The findings of the former strand of (rather slim) literature indicate that using a single instrument to address multiple externalities need not alleviate the externality being targeted (Bento et al. (2014); Landry and Bento (2013)). The latter literature however offers some support for the approach of using multiple, often complementary, policies to address a single externality. Our case is somewhere in between, with multiple externalities and a small sub-set of policies. In any case, in this setting, our study provides indicative evidence that when externalities are related, a small sub-set of instruments may be sufficient for attaining the desired environmental objectives. Evaluating whether, and when, these are also optimal is a task for future research.
References


Appendix

A Equilibrium allocation

We solve the model as a decentralized equilibrium. That is, we will assume that the representative agent in each sector maximize the corresponding objective taking prices as given.

A.1 Equilibrium conditions

There are 19 unknown quantities:


There are also 11 unknown prices:

\( p_A, p_E, p_E, p_F, p_L, p_P, r_P, r_T, p_W \) and \( p_Y \).

We thus have a total of 30 unknowns to be pinned down by the equilibrium conditions.

The representative household’s maximization problem is given in (15). Let \( \lambda \) be the multiplier on the budget constraint. The first-order conditions are

\[
\frac{\partial U}{\partial A} = \lambda p_A, \quad \frac{\partial U}{\partial F} = \lambda p_F, \quad \frac{\partial U}{\partial L_U} = \lambda p_L, \quad \frac{\partial U}{\partial T} = \lambda p_T \quad \text{and} \quad \frac{\partial U}{\partial Y} = \lambda p_Y. \tag{22}
\]

The representative energy-service producer’s maximization problem (12) gives first-order conditions

\[
p_E \frac{\partial E}{\partial E_E} = p_E, \quad p_E \frac{\partial E}{\partial A_B} = p_A \quad \text{and} \quad p_E \frac{\partial E}{\partial R} = p_R. \tag{23}
\]

The representative composite-good producer’s maximization problem is given in (14). The first-order condition is

\[
p_Y \frac{\partial Y}{\partial E_Y} = p_E. \tag{24}
\]

The representative fertilizer producer’s maximization problem (5) gives first-order conditions

\[
p_P \frac{\partial P}{\partial E_P} = p_E \quad \text{and} \quad p_P \frac{\partial P}{\partial P} = p_P. \tag{25}
\]

The representative timber producer’s maximization problem is

\[
p_T \frac{\partial T}{\partial L_T} = p_{LT}(L_T). \tag{26}
\]

[Note: Here (and in the next set of conditions), the clearing cost function is not differentiated with respect to \( L_T \) (or \( L_A \)) which reflects that the clearing costs are marginal costs that depend on the aggregate clearing.] The representative agriculture producer’s maximization problem (2) gives first-order conditions

\[
p_A \frac{\partial A}{\partial L_A} = p_{LCA}(L_A), \quad p_A \frac{\partial A}{\partial P} = p_P, \quad p_A \frac{\partial A}{\partial W} = p_W, \quad p_A \frac{\partial A}{\partial E_A} = p_E. \tag{27}
\]
Finally, we have the maximization problems representing the supply of phosphate, fish, water, fossil energy and renewable energy given in (6)-(11). The first-order conditions of these give
\[ p_P = g_P'(P), \quad p_F = g_F'(F), \quad p_W = g_W'(W), \quad p_E = (1 + \tau_E)g_E'(E) \quad \text{and} \quad p_R = g_R'(R). \] (28)

The first-order conditions (22)-(28) in total provide 21 conditions. In addition to that there are 4 market clearing conditions (1), (3), (10) and (13). Finally, the production functions in the maximization problems (2), (4), (5), (12) and (14) provide the 5 conditions required to pin down all 30 unknown quantities and prices. There is actually one more unknown, \( \lambda \) (the multiplier in the household’s maximization problem) that is pinned down by the household’s budget constraint.

### A.2 Interpretation of the equilibrium conditions

We can start by noting that the first-order conditions in (28) provide supply functions for \( P, F, W, E \) and \( R \) and the choice of cost functions \( g_X(X) \) immediately imply supply functions.

For the factors that are used in multiple sectors, the first-order conditions imply that the marginal value of using them must be the same in all sectors. In particular

- For \( A \), equations (22) and (23) imply
  \[ p_A = p\frac{\partial E}{\partial A} = \frac{1}{\lambda} \frac{\partial U}{\partial A}. \]

- For \( E \), equations (23) and (25) imply
  \[ p_E = p\frac{\partial E}{\partial E} = p\frac{\partial P}{\partial P}. \]

- For \( E \), equations (24) and (27) imply
  \[ p\varepsilon = p\frac{\partial Y}{\partial E} = p\frac{\partial A}{\partial A}. \]

- For \( L \), equations (22), (26) and (27) imply
  \[ p_L = \frac{1}{\lambda} \frac{\partial U}{\partial L_U} = p\frac{\partial T}{\partial T} \frac{1}{c_T(L_T)} = p\frac{\partial A}{\partial A} \frac{1}{c_A(L_A)}. \]

### A.3 Eliminating prices

The equilibrium can also be expressed in real terms by eliminating all prices. Equation (28) immediately gives \( p_P, p_F, p_W, p_E \) and \( p_R \) in terms of real quantities. Substituting these in (27)
gives

\[ p_A = g'_F(F) \frac{\partial U}{\partial A} \]  

(29)

Substituting \( p_R = g'_R(R) \) in (23) gives

\[ p_E = \frac{1}{\partial R} g'_R(R). \]  

(30)

Using \( p_F = g'_F(F) \) in (22) we get

\[ p_L = g'_F(F) \frac{\partial U}{\partial L}, \]  

(31)

\[ p_T = g'_F(F) \frac{\partial U}{\partial T}, \]  

(32)

and

\[ p_Y = g'_F(F) \frac{\partial U}{\partial Y}. \]  

(33)

Finally, substituting \( p_P = g'_P(P) \) in (25) gives

\[ p_P = \frac{1}{\partial P} g'_P(P) \]  

(34)

All prices are now expressed in terms of real quantities which allows for expressing all the equilibrium conditions in terms of 19 equations in 19 unknown quantities. If we abstractly denote these quantities \( Q_1, Q_2, \ldots, Q_{19} \) we can write the equilibrium conditions

\[ G_1(Q_1, Q_2, \ldots, Q_{19}; \tau) = 0 \]
\[ G_2(Q_1, Q_2, \ldots, Q_{19}; \tau) = 0 \]
\[ \vdots \]
\[ G_{19}(Q_1, Q_2, \ldots, Q_{19}; \tau) = 0, \]

(35)

where \( \tau \) is some representation of policies we want to consider (e.g., the fossil-fuel tax \( \tau_E \)).

The \( G_i \) will be given by expressions such as

\[ G_i = \frac{\partial U}{\partial Y} \frac{\partial Y}{\partial E_Y} - \frac{\partial U}{\partial A} \frac{\partial A}{\partial E_A} \]

\[ G_i = W_A + W_B - W, \]

The system (35) defines all equilibrium quantities as functions of \( \tau \). The effects of changing the policy \( \tau \) can be analyzed in (at least) two ways.

The first is to simply solve the system for all equilibrium quantities. The equilibrium prices can then be computed from the quantities. Comparing the equilibria resulting from different values of the tax then gives the effects of the tax.
The second is to trace the effects of a marginal change in \( \tau \) through the model. Fully differentiating the system (35) with respect to \( \tau \) gives a new system

\[
\frac{\partial G_1}{\partial Q_1} \frac{dQ_1}{d\tau} + \frac{\partial G_1}{\partial Q_2} \frac{dQ_2}{d\tau} + \ldots + \frac{\partial G_1}{\partial Q_{19}} \frac{dQ_{19}}{d\tau} = 0
\]

\[\vdots\]

\[
\frac{\partial G_{19}}{\partial Q_1} \frac{dQ_1}{d\tau} + \frac{\partial G_{19}}{\partial Q_2} \frac{dQ_2}{d\tau} + \ldots + \frac{\partial G_{19}}{\partial Q_{19}} \frac{dQ_{19}}{d\tau} = 0.
\]

This is a system of 19 equations in the 19 unknown \( \frac{dQ_i}{d\tau} \).

The first approach provides the most information and is also necessary to consider policy changes that are non marginal. It will, however, require fully specifying all involved functions (choosing the functional forms and setting numerical values to all parameters in these).

The second approach boils down to solving a linear system of equations. The numerical values required to derive the effects of a policy change are the coefficients \( \frac{\partial G_i}{\partial Q_j} \). This is potentially a significantly smaller set of values and it is possible to break them down into components (e.g., elasticities and cost shares) that have been previously estimated.

### A.4 Equilibrium conditions without prices

We will here derive all equilibrium conditions expressed in quantities. There are no prices in the 4 market-clearing conditions (1), (3), (10) and (13). The same holds for the production functions in the maximization problems (2), (4), (5), (12) and (14).

Substituting for the prices in (23) gives

\[
\frac{1}{g_R(R)} \frac{\partial \mathcal{E}}{\partial R} = \frac{1}{(1 + \tau E)g'_E(E)} \frac{\partial \mathcal{E}}{\partial \mathcal{E}}
\]

\[
\frac{1}{g'_R(R)} \frac{\partial \mathcal{E}}{\partial R} = \frac{1}{g'_W(W)} \frac{\partial \mathcal{E}}{\partial W} \frac{\partial \mathcal{A}}{\partial A_B}
\]

Substituting for prices in (24) gives

\[
\frac{1}{g'_F(F)} \frac{\partial U}{\partial F} = \frac{1}{g_R(R)} \frac{\partial \mathcal{E}}{\partial R} \frac{\partial Y}{\partial Y} \frac{\partial U}{\partial Y}
\]

Substituting for prices in (26)

\[
\frac{\partial U}{\partial T} \frac{\partial T}{\partial L_T} = \frac{\partial U}{\partial L_U} c_T(L_T).
\]
Substituting for prices in (27) gives

\[ \frac{dU}{dA} \frac{dA}{dL_A} = \frac{dU}{dL_U} c_A(L_A) \]

\[ \frac{1}{g_F'(F)} \frac{dF}{dF} = \frac{1}{g_W'(W)} \frac{dA}{dW} \]

\[ \frac{1}{g_W'(W)} \frac{dW}{dW} = \frac{1}{g_F'(F)} \frac{dA}{dP} \]

\[ \frac{1}{g_W'(W)} \frac{dW}{dW} = \frac{1}{g_F'(F)} \frac{dA}{dE} \]

Substituting for prices in first condition of (25) gives

\[ \frac{1}{1 + \tau_E} \frac{dP}{dP} = \frac{1}{g_F'(F)} \frac{dA}{dP} \]

Finally, the budget constraint can be rewritten in terms of only quantities

\[ I = \frac{g_F'(F)}{g_U'(U)} \left[ \frac{dU}{dA} A + \frac{dU}{dY} Y + \frac{dU}{dP} P + \frac{dU}{dL} L + \frac{dU}{dT} T \right] \]

**B The effect of a carbon tax**

As described above, we can treat all equilibrium quantities as implicit functions of \( \tau_E \) and differentiate the equilibrium conditions fully with respect to \( \tau_E \). This will give us system of 19 linear equations in the changes in the quantities \( X \).

It is more convenient to work with relative changes in quantities and we denote the relative change in quantity \( X \) by

\[ \hat{X} \equiv \frac{1}{X} \frac{dX}{d\tau_E} \]

Good \( X \in \{ E, F, P, R, W \} \) is extracted at cost \( g_X(X) \). This implies supply functions (28).

We can differentiate fully with respect to the price \( p_X \) to get

\[ 1 = g''_X(X) \frac{dX}{dp_X} \Rightarrow \frac{p_X}{X} \frac{dX}{dp_X} = \frac{g''_X(X)}{g'_X(X)} \frac{1}{X} \]

This is the inverse of the supply elasticity of good \( X \) and we use notation

\[ \Lambda_X \equiv \frac{g''_X(X)}{g'_X(X)} X \]

Consider now production or utility function \( Z \in \{ A, E, P, T, U, Y \} \). Denote the set of its
arguments by \( J_Z \). That is
\[
\begin{align*}
J_A &\equiv \{L_A, P, W, E_A\} \\
J_P &\equiv \{E_P, P\} \\
J_E &\equiv \{A_B, E_F, R\} \\
J_U &\equiv \{Y, A_F, F, L_U, T\}.
\end{align*}
\]

[There are also the single-argument functions \( T \) and \( Y \)]

For function \( Z \) and input \( X \in J_Z \) we can define elasticities
\[
\Gamma_Z^X \equiv \frac{X \partial Z}{Z \partial X}. \tag{37}
\]

For function \( Z \) and arguments \( X_1, X_2 \in J_Z \) we can define elasticities
\[
\Gamma_{X_1, X_2}^Z \equiv \frac{\partial^2 Z}{\partial X_1 \partial X_2}. \tag{38}
\]

### B.1 Comparative statics

We start by differentiating the market-clearing conditions:
\[
\begin{align*}
(1) \Rightarrow & 0 = L_A \dot{L}_A + L_T \dot{L}_T + L_U \dot{L}_U \\
(10) \Rightarrow & \dot{E} = \frac{E_\xi}{E} \ddot{E}_\xi + \frac{E_P}{E} \ddot{E}_P \\
(3) \Rightarrow & \dot{A} = \frac{A_B}{A} \dot{A}_B + \frac{A_F}{A} \dot{A}_F \\
(13) \Rightarrow & \dot{\xi} = \frac{\xi_A}{\xi} \ddot{\xi}_A + \frac{\xi_Y}{\xi} \ddot{\xi}_Y \\
(2) \Rightarrow & \dot{\xi} = \Gamma_A^\xi \dot{\xi}_A + \Gamma_B^\xi \dot{\xi}_B + \Gamma_F^\xi \dot{\xi}_F + \Gamma_E^\xi \dot{\xi}_E + \Gamma_Y^\xi \dot{\xi}_Y \tag{43}
\end{align*}
\]

Similarly, differentiating the production functions gives
\[
\begin{align*}
(4) \Rightarrow & \dot{T} = \Gamma_T^T \dot{T}_T \\
(5) \Rightarrow & \dot{\xi} = \Gamma_{E_P}^\xi \dot{E}_P + \Gamma_E^\xi \ddot{E}_E + \Gamma_R^\xi \ddot{R} = \sum_{X \in J_P} \Gamma_X^\xi \dot{X} \\
(12) \Rightarrow & \dddot{\xi} = \Gamma_{A_B}^\xi \dddot{A}_B + \Gamma_{E_F}^\xi \dddot{E}_F + \Gamma_F^\xi \dddot{F} + \Gamma_E^\xi \dddot{E}_E + \Gamma_Y^\xi \dddot{Y} = \sum_{X \in J_F} \Gamma_X^\xi \dddot{X} \\
(14) \Rightarrow & \dot{\xi} = \Gamma_{\xi_Y}^Y \dot{\xi}_Y \tag{46}
\end{align*}
\]
We now differentiate the remaining first-order conditions and simplify.

\[
\sum_{X \in \mathcal{I}_e} (\Gamma^e_{R,X} - \Gamma^e_{E_r,X}) \, \hat{X} = \Lambda R \hat{R} - \frac{1}{1 + \tau_E} - \Lambda E \hat{E}
\]

(48)

\[
\sum_{X \in \mathcal{I}_e} (\Gamma^e_{R,X} - \Gamma^e_{A_r,X}) \, \hat{X} = \sum_{X \in \mathcal{I}_A} \Gamma^A_{W,X} \hat{X} + \Lambda R \hat{R} - \Lambda W \hat{W}
\]

(49)

\[
\sum_{X \in \mathcal{I}_u} (\Gamma^U_{F,X} - \Gamma^U_{Y,X}) \, \hat{X} = \sum_{X \in \mathcal{I}_A} \Gamma^A_{W,X} \hat{X} + \Lambda F \hat{F} - \Lambda R \hat{R}
\]

(50)

\[
\sum_{X \in \mathcal{I}_u} (\Gamma^U_{F,X} - \Gamma^U_{A_r,X}) \, \hat{X} = \sum_{X \in \mathcal{I}_A} \Gamma^A_{W,X} \hat{X} + \Lambda F \hat{F} - \Lambda W \hat{W}
\]

(51)

\[
\sum_{X \in \mathcal{I}_u} (\Gamma^A_{W,X} - \Gamma^A_{P_r,X}) \, \hat{X} = \Lambda W \hat{W} - \Lambda P \hat{P} + \sum_{X \in \mathcal{I}_P} \Gamma^P_{P,X}
\]

(52)

\[
\sum_{X \in \mathcal{I}_u} (\Gamma^A_{W,X} - \Gamma^A_{L_r,X}) \, \hat{X} = \Lambda W \hat{W} - \Lambda R \hat{R} + \sum_{X \in \mathcal{I}_A} \Gamma^A_{L,A,X} \hat{X}
\]

(53)

\[
\sum_{X \in \mathcal{I}_u} (\Gamma^U_{T,X} - \Gamma^U_{L_u,X}) \, \hat{X} = \frac{1}{1 + \tau_E} + \Lambda E \hat{E} - \Lambda P \hat{P}
\]

(54)

\[
\sum_{X \in \mathcal{I}_u} (\Gamma^U_{T,X} - \Gamma^U_{L_u,X}) \, \hat{X} = \frac{L_T}{C_T} \frac{dT}{dL_T} - \Gamma^T_{L_T,L_T} \hat{L}_T
\]

(55)

\[
\sum_{X \in \mathcal{I}_u} (\Gamma^U_{A,X} - \Gamma^U_{L_u,X}) \, \hat{X} = \frac{L_A}{C_A} \frac{dC_A}{dL_A} \hat{L}_A - \sum_{X \in \mathcal{I}_A} \Gamma^A_{L,A,X} \hat{X}
\]

(56)

The budget constraint can be rewritten as

\[
\frac{\partial U}{\partial P} = \left[ \frac{\partial U}{\partial A_F} \frac{dA_F}{dF} \right] + \frac{\partial U}{\partial Y} Y + \frac{\partial U}{\partial F} F + \frac{\partial U}{\partial L_U} L_U + \frac{\partial U}{\partial T} T
\]

We can define the expenditure share of consumption good \( Z \) as

\[
s_Z = \frac{\partial U}{\partial X} Z
\]

Differentiating the budget constraint fully (and assuming that the income is unchanged) now gives

\[
\sum_{Z \in \mathcal{I}_v} \Gamma^U_{F,Z} \hat{Z} - \Lambda F \hat{F} = \sum_{Z \in \mathcal{I}_v} s_Z \left[ \hat{Z} + \sum_{X \in \mathcal{I}_v} \Gamma^U_{Z,X} \hat{X} \right]
\]

\[
= \sum_{Z \in \mathcal{I}_v} s_Z \hat{Z} + \sum_{Z \in \mathcal{I}_v} s_Z \sum_{X \in \mathcal{I}_v} \Gamma^U_{Z,X} \hat{X}
\]

\[
= \sum_{Z \in \mathcal{I}_v} \left[ s_Z + \sum_{X \in \mathcal{I}_v} s_X \Gamma^U_{X,Z} \right] \hat{Z}
\]

38
\[ \Lambda_F \hat{F} = \sum_{Z \in I_U} \left[ \Gamma^{Z}_{FZ} - s_Z - \sum_{X \in I_U} s_X \Gamma^{Z}_{XZ} \right] \hat{Z} \]  

(57)

B.2 Examples of functional forms

We defined various combinations of derivatives in equations (36), (37) and (38). We will here derive what these are for some standard functional forms.

We start with (36). If we assume cost functions of the form

\[ g(X) = \mu X^{\alpha_X + 1} \]  

(58)

we get elasticity

\[ \Lambda_X = \frac{g''(X)}{g'(X)} X = \alpha_X. \]  

(59)

For (37) and (38) we need to specify production and utility functions. We assume that both of these are (potentially nested) CES functions. We start by assuming a production or utility function

\[ Z(X_1, \ldots, X_J) = \left[ \sum_{j=1}^{J} \gamma_j X_j^{\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}. \]  

(60)

The expression for \( \Gamma^Z_X \) is

\[ \Gamma^Z_{X_j} = \frac{X_j \partial Z}{Z \partial X_j} \]  

(61)

The first-order derivatives are:

\[ \frac{\partial Z}{\partial X_j} = \gamma_j \left( \frac{Z}{X_j} \right)^{\frac{1}{\sigma}} \]  

(62)

The expression for \( \Gamma^Z_{X_i, X_k} \) requires the second-order derivatives:

\[ \frac{\partial^2 Z}{\partial X_k \partial X_i} = \frac{\partial Z}{\partial X_i} \left[ \frac{1}{\sigma} \gamma_i Z \frac{\partial}{\partial X_i} - \frac{1}{\sigma} X_i \right] = \frac{\partial Z}{\partial X_i} \frac{1}{\sigma} \left[ \frac{1}{Z \partial X_i} X_i - 1 \right] \frac{1}{X_i} \]  

(63)

\[ \frac{\partial^2 Z}{\partial X_k \partial X_i} = \frac{\partial Z}{\partial X_i} \frac{1}{\sigma} \frac{\partial Z}{\partial X_k}. \]  

(64)

We now have

\[ \Gamma^Z_{X_i, X_k} = \begin{cases} -\frac{1}{\sigma} \left[ 1 - \frac{1}{Z \partial X_i} X_i \right] & \text{if } k = i \\ \frac{1}{\sigma} \frac{\partial Z}{\partial X_k} & \text{if } k \neq i \end{cases} \]  

(65)

Note that if prices reflect marginal product or marginal utility then

\[ \frac{1}{Z \partial X_j} X_j = \Gamma^Z_{X_j}, \]  

(66)

is the cost share of \( X_j \) (out of total expenditures on consumption goods or inputs).
With
\[ P = \left[ \gamma_{P,E} \frac{e^{P}}{P} + \gamma_{P,P} P \right] \frac{e^{P}}{P} \]  
and
\[ E = \left[ \gamma_{E,A} A \frac{e^{E}}{E} + \gamma_{E,E} E \frac{e^{E}}{E} + \gamma_{E,R} R \frac{e^{E}}{E} \right] \frac{e^{E}}{E} , \]
this implies that
\[ \sum_{X \in P} \left[ \Gamma_{P,E,X}^P - \Gamma_{P,X}^P \right] \hat{X} = \frac{1}{\sigma_P^2} \left[ \hat{P} - \hat{E}_P \right] \quad (67) \]
\[ \sum_{X \in E} \left[ \Gamma_{E,E,X}^E - \Gamma_{E,X}^E \right] \hat{X} = \frac{1}{\sigma_E^2} \left[ \hat{E}_E - \hat{R} \right] \quad (68) \]
\[ \sum_{X \in E} \left[ \Gamma_{E,R,X}^E - \Gamma_{E,X}^E \right] \hat{X} = \frac{1}{\sigma_E^2} \left[ \hat{A}_B - \hat{R} \right] . \quad (69) \]
\[ \sum_{X \in E} \left[ \Gamma_{E,R,X}^E - \Gamma_{E,A} A \right] \hat{X} = \frac{1}{\sigma_E^2} \left[ \hat{A}_B - \hat{R} \right] . \quad (70) \]
We could also have a nested CES function so that
\[ X_1 = \left[ \gamma_{1,1} X_{1,1}^{1-\gamma_{1,1}} + \gamma_{1,2} X_{1,2}^{1-\gamma_{1,2}} \right] \frac{e^{X_1}}{X_1} . \quad (71) \]
The first-order derivative is
\[ \frac{\partial Z}{\partial X_{1,j}} = \frac{\partial Z}{\partial X_1} \frac{\partial X_1}{\partial X_{1,j}} = \gamma_1 \left( Z \right) \frac{1}{X_1} \gamma_{1,j} \left( \frac{X_1}{X_{1,j}} \right) \frac{1}{X_{1,j}} . \]
We also have
\[ \Gamma_{X_{1,j}}^Z = \frac{X_{1,j}}{Z} \frac{\partial Z}{\partial X_{1,j}} = \frac{X_1}{Z} \frac{\partial Z}{\partial X_1} \frac{\partial X_1}{\partial X_{1,j}} = \Gamma_{X_1} \Gamma_{X_{1,j}}^X . \]
The second-order derivatives are
\[ \frac{\partial^2 Z}{\partial X_{1,j}^2} = \frac{\partial Z}{\partial X_{1,j}} \left[ \frac{1}{\sigma} \left( \Gamma_{X_1}^Z - 1 \right) \Gamma_{X_{1,j}}^X + \frac{1}{\sigma_1} \left( \Gamma_{X_{1,j}}^Z - 1 \right) \Gamma_{X_1}^X \right] \frac{1}{X_{1,j}} \]
\[ \frac{\partial^2 Z}{\partial X_{1,i} \partial X_{1,j}} = \frac{\partial Z}{\partial X_{1,j}} \left[ \frac{1}{\sigma} \left( \Gamma_{X_1}^Z - 1 \right) + \frac{1}{\sigma_1} \Gamma_{X_{1,j}}^X \Gamma_{X_1}^X \right] \frac{1}{X_{1,i}} \]
for \( i \neq j \)
\[ \frac{\partial^2 Z}{\partial X_{k} \partial X_{1,j}} = \frac{\partial Z}{\partial X_{1,j}} \frac{\gamma_{k}}{X_k} \frac{1}{X_{1,j}} \quad \text{for} \quad k \neq 1 . \]
The first-order derivatives and second-order derivatives with respect to \( j \neq 1 \) and \( k \neq 1 \) are the same as before. Considering a \( k \neq 1 \) we get the cross derivative
\[ \frac{\partial^2 Z}{\partial X_{k} \partial X_{1,j}} = \frac{\partial Z}{\partial X_{k}} \frac{1}{\sigma} \Gamma_{X_{1,j}}^X \frac{1}{X_{1,j}} . \quad (73) \]
We can now compute
\[
\Gamma_{X_1,j,X_1,j}^Z = \frac{1}{\sigma} (\Gamma_{X_1}^Z - 1) \Gamma_{X_1,j}^X + \frac{1}{\sigma_1} (\Gamma_{X_1}^X - 1)
\]
\[
\Gamma_{X_1,i,X_1,j}^Z = \frac{1}{\sigma} (\Gamma_{X_1}^Z - 1) + \frac{1}{\sigma_1} \Gamma_{X_1,j}^X \text{ for } i \neq j
\]
\[
\Gamma_{X_k,i,X_1,j}^Z = \frac{1}{\sigma} \Gamma_{X_1}^Z \text{ for } k \neq 1.
\]
\[
\Gamma_{X_1,i,X_1,j}^Z = \frac{1}{\sigma} \Gamma_{X_k}^Z \text{ for } k \neq 1.
\]

One interesting consequence is that \(\Gamma_{X_1,j,X_1,j}^Z\) are independent of \(j\) if \(k \neq 1\).

If we assume a nested CES utility function
\[
U(F, \bar{F}) = \left[ \gamma_{U, \bar{F}} \bar{F}^{\frac{\sigma_{U-1}}{\sigma_{U}}} + \gamma_{U,F} F^{\frac{\sigma_{U-1}}{\sigma_{U}}} \right]^{\frac{\sigma_{U}}{\sigma_{U}-1}}
\]

where
\[
F = \left[ \gamma_{F, \bar{A}} \bar{A}^{\frac{\sigma_{F-1}}{\sigma_{F}}} + \gamma_{F,A} A^{\frac{\sigma_{F-1}}{\sigma_{F}}} \right]^{\frac{\sigma_{F}}{\sigma_{F}-1}},
\]
\[
\bar{F} = \left[ \gamma_{\bar{F},Y} Y^{\frac{\sigma_{\bar{F}-1}}{\sigma_{\bar{F}}} \bar{F}^{\frac{\sigma_{\bar{F}-1}}{\sigma_{\bar{F}}}}} + \gamma_{\bar{F},L_U} L_U^{\frac{\sigma_{\bar{F}-1}}{\sigma_{\bar{F}}}} + \gamma_{\bar{F},T} T^{\frac{\sigma_{\bar{F}-1}}{\sigma_{\bar{F}}}} \right]^{\frac{\sigma_{\bar{F}}}{\sigma_{\bar{F}}-1}}
\]

Then the right-hand side of the above comparative static conditions become
\[
\sum_{X \in \mathcal{I}_U} (\Gamma_{U,F,X}^U - \Gamma_{U,Y,X}^U) \bar{X} = \left[ \frac{1}{\sigma_{U}} - \frac{1}{\sigma_{\bar{F}}} \right] \Gamma_{\bar{Y}}^U + \frac{1}{\sigma_{\bar{F}}} \bar{Y} - \left( \frac{1}{\sigma_{U}} - \frac{1}{\sigma_{\bar{F}}} \right) \Gamma_{\bar{F},A}^U \bar{A}_X
\]
\[
- \left[ \frac{1}{\sigma_{U}} - \frac{1}{\sigma_{\bar{F}}} \right] \Gamma_{\bar{F}}^U + \frac{1}{\sigma_{\bar{F}}} \bar{F} + \left( \frac{1}{\sigma_{U}} - \frac{1}{\sigma_{\bar{F}}} \right) \Gamma_{L_U}^U \bar{L}_U
\]
\[
+ \left( \frac{1}{\sigma_{U}} - \frac{1}{\sigma_{\bar{F}}} \right) \Gamma_{\bar{T}}^U \bar{T}
\]

Likewise, we assume that the agricultural production function is a nested CES function
\[
A(\bar{L}_A, \bar{L}_A) = \left[ \gamma_{A, \bar{L}_A} \bar{L}_A^{\frac{\sigma_{A-1}}{\sigma_{A}}} + \gamma_{A,L_A} L_A^{\frac{\sigma_{A-1}}{\sigma_{A}}} \right]^{\frac{\sigma_{A}}{\sigma_{A}-1}}
\]

where
\[
\bar{L}_A = \left[ \gamma_{\bar{L}_A} P^{\frac{\sigma_{\bar{L}_A-1}}{\sigma_{\bar{L}_A}}} + \gamma_{\bar{L}_A,W} W^{\frac{\sigma_{\bar{L}_A-1}}{\sigma_{\bar{L}_A}}} + \gamma_{\bar{L}_A,E} E^{\frac{\sigma_{\bar{L}_A-1}}{\sigma_{\bar{L}_A}}} \right]^{\frac{\sigma_{\bar{L}_A}}{\sigma_{\bar{L}_A}-1}}.
\]
\[
\sum_{X \in J_A} (\Gamma_{W,X}^A - \Gamma_{P,X}^A) \tilde{X} = \frac{1}{\sigma_{L_A}} (\tilde{P} - \tilde{W})
\]
\[
\sum_{X \in J_A} (\Gamma_{W,X}^A - \Gamma_{\tilde{E}_A,X}^A) \tilde{X} = \frac{1}{\sigma_{L_A}} (\tilde{\epsilon}_A - \tilde{W})
\]

### B.3 Comparative statics with CES functions

The conditions derived from market clearing are unaffected:

1. \(0 = \frac{L_A}{L} \tilde{L}_A + \frac{L_T}{L} \tilde{L}_T + \frac{L_U}{L} \tilde{L}_U\) (78)
2. \(\tilde{\epsilon} = \frac{E_e}{E} \tilde{E}_e + \frac{E_p}{E} \tilde{E}_p\) (79)
3. \(\tilde{A} = \frac{A}{\tilde{A}} A_B + \frac{A}{\tilde{A}} A_F\) (80)
4. \(\tilde{E} = \frac{E_A}{\tilde{E}} A_A + \frac{E_Y}{\tilde{E}} A_Y\) (81)

The conditions derived by differentiating the production functions are also unchanged

1. \(\tilde{A} = \Gamma_{L_A}^A \tilde{L}_A + \Gamma_{L_T}^A \tilde{L}_T + \Gamma_{L_U}^A \tilde{L}_U + \Gamma_{E_A}^A \tilde{E}_A = \sum_{X \in J_A} \Gamma_X^A \tilde{X}\) (82)
2. \(\tilde{T} = \Gamma_{E_T}^T \tilde{E}_T\) (83)
3. \(\tilde{P} = \Gamma_{E_P}^P \tilde{E}_P + \Gamma_{E_P}^P \tilde{E}_P = \sum_{X \in J_P} \Gamma_X^P \tilde{X}\) (84)
4. \(\tilde{\epsilon} = \Gamma_{A_B}^E \tilde{A}_B + \Gamma_{E_X}^E \tilde{E}_X + \Gamma_{R_X}^E \tilde{R} = \sum_{X \in J_E} \Gamma_X^E \tilde{X}\) (85)
5. \(\tilde{Y} = \Gamma_{E_Y}^Y \tilde{E}_Y\) (86)
Some of the remaining conditions can be simplified and the resulting conditions are

\[
\frac{1}{\sigma_e} \left[ \hat{E}_e - \bar{R} \right] = \lambda_R \hat{R} - \frac{1}{1 + \tau_e} - \Lambda_E \hat{E} \tag{87}
\]

\[
\frac{1}{\sigma_e} \left[ \hat{A}_B - \bar{R} \right] = \sum_{X \in I_A} \Gamma_{W,X}^1 \hat{X} + \Lambda_R \hat{R} - \Lambda_W \hat{W} \tag{88}
\]

\[
\sum_{X \in I_U} \left( \Gamma_{F,X}^U - \Gamma_{Y,X}^U \right) \hat{X} = \frac{1}{\sigma_e} \left[ \sum_{X \in I_e} \Gamma_{X}^e \hat{X} - \bar{R} \right] + \gamma_{Y,\bar{X},\bar{X}} \hat{Y} + \Lambda_F \hat{F} - \Lambda_R \hat{R} \tag{89}
\]

\[
\frac{1}{\sigma_F} \left( \hat{A}_F - \hat{F} \right) = \sum_{X \in I_A} \Gamma_{W,X}^1 \hat{X} + \Lambda_F \hat{F} - \Lambda_W \hat{W} \tag{90}
\]

\[
\frac{1}{\sigma_{L_A}} \left[ \hat{P} - \bar{W} \right] = \Lambda_W \hat{W} - \Lambda_P \hat{P} + \frac{1}{\sigma_P} \left[ \sum_{X \in I_P} \Gamma_{X}^P \hat{X} - \bar{P} \right] \tag{91}
\]

\[
\frac{1}{\sigma_{L_A}} \left[ \hat{E}_A - \bar{W} \right] = \Lambda_W \hat{W} - \Lambda_R \hat{R} + \frac{1}{\sigma_e} \left[ \sum_{X \in I_e} \Gamma_{X}^e \hat{X} - \bar{R} \right] \tag{92}
\]

\[
\frac{1}{\sigma_F} \left[ \hat{P} - \bar{F}_P \right] = \frac{1}{1 + \tau_e} + \lambda_E \hat{E} - \lambda_F \hat{F} \tag{93}
\]

\[
\frac{1}{\sigma_{L_A}} \left[ \hat{L}_U - \bar{T} \right] = \left[ \frac{L_T dC_T}{C_T dL_T} - \Gamma_{L_T,L_T}^T \right] \hat{L}_T \tag{94}
\]

\[
\sum_{X \in I_U} \left( \Gamma_{A_F,X}^U - \Gamma_{L_U,X}^U \right) \hat{X} = \frac{L_A dC_A}{C_A dL_A} \hat{L}_A - \sum_{X \in I_A} \Gamma_{L_A,X}^A \hat{X} \tag{95}
\]

\[
\Lambda_F \hat{F} = \sum_{Z \in I_U} \left[ \Gamma_{F,Z}^U - s_Z - \sum_{X \in I_U} s_X \Gamma_{X,Z}^U \right] \hat{Z} \tag{96}
\]

With CES utility, we further have

\[
\sum_{X \in I_U} s_X \Gamma_{X,Z}^U = 0 \quad \text{and} \quad s_Z = \Gamma_{Z}^U.
\]

The last condition then becomes

\[
\Lambda_F \hat{F} = \sum_{Z \in I_U} \left[ \Gamma_{F,Z}^U - \Gamma_{Z}^U \right] \hat{Z} \tag{96}
\]

For \( Z \in \{Y, L_U, T\} \) we have

\[
\Gamma_{F,Z}^U - \Gamma_Z^U = \frac{1 - \sigma_U}{\sigma_U} \Gamma_Z^U.
\]

For the different food sources we have

\[
\Gamma_{F,A_F}^U - \Gamma_{A_F}^U = \frac{1 - \sigma_U}{\sigma_U} \Gamma_{A_F}^U + \left( \frac{1}{\sigma_F} - \frac{1}{\sigma_U} \right) \Gamma_{A_F}^U
\]
and
\[ \Gamma^U_{F,F} - \Gamma^U_F = \frac{1 - \sigma_U}{\sigma_U} \Gamma^U_F + \left( \frac{1}{\sigma_F} - \frac{1}{\sigma_U} \right) \Gamma^X_F - \frac{1}{\sigma_F}. \]

### B.3.1 Case with fixed land use in agriculture

If we impose a lower bound on land use in agriculture (and assume it is binding), this is equivalent to fixing \( L_A \) in the equations above. This corresponds to excluding \( \hat{L}_A \) from all the comparative-statics conditions above and removing the condition derived from the first-order condition with respect to \( L_A \).

### B.4 Price changes

When solving the model, we eliminated all prices from the equilibrium conditions. We will here derive expressions for the changes in prices resulting from a change in the carbon tax. As for quantities we denote relative price changes by

\[ \hat{p}_X = \frac{1}{p_X} \frac{dp_X}{d\tau_E}. \]  \hfill (97)

The endogenous prices are \( p_A, p_F, p_P, p_L, p_P, p_R, p_T, p_W \) and \( p_Y \). For \( X \in \{P,F,W,R\} \), the prices \( p_X \) are given in (28) and the induced price changes are

\[ \frac{dp_X}{d\tau_E} = g'_X(X) \frac{dX}{d\tau_E} \Rightarrow \hat{p}_X = \frac{g'_X(X)X}{g(X)} \hat{X} = \{36\} = \Lambda_X \hat{X}. \]  \hfill (98)

The (buyer) price of fossil fuel, \( p_E \), is similar except that it also contains the carbon tax directly. We get

\[ \hat{p}_E = \frac{1}{1 + \tau_E} + \Lambda_E \hat{E}, \]  \hfill (99)

where the first term captures the direct effect of the tax while the second term captures the change in the producer price.

The change in the price of energy services can be derived from (23):

\[ \hat{p}_E = \Lambda_R \hat{R} - \sum_{X \in \{P,F,W\}} \Gamma^F_{R,X} \hat{X} = (\Lambda_R - \Gamma^F_{R,R}) \hat{R} - \Gamma^F_{R,A} \hat{A} - \Gamma^F_{R,E} \hat{E}. \]  \hfill (100)

The change in the price of fertilizers can be derived from (34)

\[ \hat{p}_P = (\Lambda_P - \Gamma^P_{P,P}) \hat{P} - \Gamma^P_{P,E} \hat{E}. \]  \hfill (101)

The change in the price of the composite consumption good can be derived from (24)

\[ \hat{p}_Y = \hat{p}_E - \Gamma^Y_{E_e} \hat{E}_Y = (\Lambda_R - \Gamma^Y_{R,R}) \hat{R} - \Gamma^Y_{R,A} \hat{A} - \Gamma^Y_{R,E} \hat{E} - \Gamma^Y_{E_e} \hat{E}_Y. \]  \hfill (102)
The change in the price of agricultural production can be derived from (27)

\[ \hat{p}_A = (\Lambda W - \Gamma_{W,W}^A) \hat{W} - \Gamma_{W,L_A}^A \hat{L}_A - \Gamma_{W,F}^A \hat{P} - \Gamma_{W,\xi_A}^A \hat{\xi}_A. \]  

(103)

The change in the price of land can be derived from (27)

\[
\hat{p}_L = \hat{p}_A + \sum_{X \in \xi_A} \Gamma_{L_A,X}^A \hat{X} - \hat{V}_A \hat{L}_A
- \Lambda W \hat{W} + \sum_{X \in \xi_A} (\Gamma_{L_A,X}^A - \Gamma_{W,X}^A) \hat{X} - \hat{V}_A \hat{L}_A
= (\Lambda W + \Gamma_{L_A,W}^A - \Gamma_{W,W}^A) \hat{W} + (\Gamma_{L_A,L_A}^A - \Gamma_{W,L_A}^A) \hat{L}_A
+ (\Gamma_{L_A,P}^A - \Gamma_{W,P}^A) \hat{P} + (\Gamma_{L_A,\xi_A}^A - \Gamma_{W,\xi_A}^A) \hat{\xi}_A - \hat{V}_A \hat{L}_A. \]

(104)

When land use in agriculture is restricted we can instead compute the change in the price of land in the other uses from (31)

\[
\hat{p}_L = \Lambda_F \hat{F} + \sum_{X \in \xi_U} (\Gamma_{L_U,X}^U - \Gamma_{F,X}^U) \hat{X}
= (\Lambda_F + \Gamma_{L_U,F}^U - \Gamma_{F,F}^U) \hat{F} + (\Gamma_{L_U,Y}^U - \Gamma_{F,Y}^U) \hat{Y} + (\Gamma_{L_U,A_F}^U - \Gamma_{F,A_F}^U) \hat{A}_F
+ (\Gamma_{L_U,L_U}^U - \Gamma_{F,L_U}^U) \hat{L}_U + (\Gamma_{L_U,T}^U - \Gamma_{F,T}^U) \hat{T} \]

(105)

The change in the price of timber production can be derived from (26)

\[ \hat{p}_T = \hat{p}_L + (V_T - \Gamma_{T,T}^T) \hat{L}_T. \]

(106)

Note that this is potentially problematic when fixing land. Then we can instead use (32) to get

\[ \hat{p}_T = \Lambda_F \hat{F} + \sum_{X \in \xi_U} (\Gamma_{T,X}^U - \Gamma_{F,X}^U) \hat{X}. \]

(107)

We can also define a price of food \( p_F \) as

\[ p_F \equiv \frac{p_F F + p_A A_F}{F}. \]

The change in this price is

\[
\hat{p}_F = \frac{p_F F}{p_F F + p_A A_F} \left( \hat{F} + \hat{p}_F \right)
+ \frac{p_F A_F}{p_F F + p_A A_F} \left( \hat{A}_F + \hat{p}_A \right)
- \frac{F}{\hat{F} \partial F} \hat{F}
- \frac{A_F}{\hat{F} \partial A_F} \hat{A}_F. \]

Noting that

\[
\frac{p_F F}{p_F F + p_A A_F} = \frac{F}{\hat{F}} \partial F = \Gamma_F^F \quad \text{and} \quad \frac{p_A A_F}{p_F F + p_A A_F} = \frac{A_F}{\hat{F}} \partial A_F = \Gamma_{A_F}. \]
this simplifies to
\[
\hat{\rho}_F = \Gamma_F^F \hat{\rho}_F + \Gamma_{A,F}^F \hat{\rho}_A. \tag{108}
\]

B.5 Calibration

The next section presents the factor shares used to calibrate the model and the remaining needed parameters are given (including their sources) in table 2.

B.5.1 Factor/expenditure shares

We treat capital and labor as fixed. The following shares are computed from the GTAP database (Aguiar et al., 2016) and summarized in table 1.

**Agriculture** In the agricultural production function, we assume that labor and capital are part of the non-land composite. The factor share of land is 19.2%. The factors shares of fertilizers, water and energy are 6.43, 1.93 and 3.33 respectively. Their respective shares out of non-land inputs are their total shares divided by the total non-land share. This means that \(\Gamma_{L_A}^A = 0.192\), \(\Gamma_{F_A}^A = 0.808\), \(\Gamma_{A_L}^A = 0.0643\), \(\Gamma_{L_A}^F = 0.808\), \(\Gamma_{F_A}^F = 0.0796\), \(\Gamma_{E_A}^F = 0.0239\) and \(\Gamma_{E_A}^A = 0.0412\).

**Energy services** The shares of biofuels, fossil fuels and renewables are 0.37%, 94.33% and 5.30% respectively. That is \(\Gamma_{E_B}^E = 0.0037\), \(\Gamma_{E_F}^E = 0.9433\) and \(\Gamma_{E_R}^E = 0.0530\).

**Utility** The expenditure shares of food from agriculture, fish, manufactured goods, recreational land use and timber are 11.93%, 0.42%, 86.86%, 0.15% and 0.65%. This gives first nest level expenditure shares \(\Gamma_{A_F}^U = 0.1235\) and \(\Gamma_{F_A}^U = 0.8765\). The second level expenditure shares are \(\Gamma_{A_F}^E = 0.9660\), \(\Gamma_{F_A}^E = 0.0340\), \(\Gamma_{F_E}^U = 0.9910\), \(\Gamma_{L_U}^E = 0.001711\) and \(\Gamma_{T}^E = 0.007416\).

**Timber** We here assume a Cobb-Douglas function with income share of land equal to 37.48%. That is \(\Gamma_{L_T}^T = 0.3748\).

**Composite good** We here assume a Cobb-Douglas production function with cost share of energy equal to 6.38%. that is, \(\Gamma_{E}^Y = 0.0638\).

**Fertilizers** The factor share of energy is 10.95%. The factor share of phosphate is assumed to a share \(\xi_P = 0.5\) out of the factor share of non energy intermediates 62.53%. That is \(\Gamma_P^E = 0.1095\) and \(\Gamma_P^P = 0.5 * 0.6253 = 0.3127\).
<table>
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<tr>
<th>Factor share</th>
<th>value</th>
<th>comment</th>
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<td>$-\Gamma^L_{EA}$</td>
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<td>$\Gamma^E_{PA}$</td>
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<td>$\Gamma^F_{EA}$</td>
<td>94.33%</td>
<td>$(1 - \Gamma^F_{A})$</td>
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Table 1: Parameters – Factor Shares
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<td>$\sigma_F$</td>
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<td>$\sigma_{nF}$</td>
<td>Drawn from estimates of substitutability derived from Drupp (2018)</td>
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<tr>
<td>$\sigma_A$</td>
<td>Steinbuks and Hertel (2016)</td>
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</tr>
<tr>
<td>$\sigma_P$</td>
<td>Assumed based on reading of the literature</td>
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<td>$\sigma_{nLA}$</td>
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</tr>
<tr>
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</tr>
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<tr>
<td>$\Lambda_E$</td>
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</tr>
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</tr>
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<td>$\Lambda_{Pbo}$</td>
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<td>$V_T$</td>
<td>Based on Steinbuks and Hertel (2016)</td>
<td>0.05</td>
</tr>
<tr>
<td>$V_A$</td>
<td>Based on Steinbuks and Hertel (2016)</td>
<td>0.05</td>
</tr>
<tr>
<td>$\tau_E$</td>
<td>A global carbon tax currently does not exist.</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Parameters – Elasticities and Quantities