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Environment, Uncertainty, and Option Values

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Environment, Uncertainty, and Option Values[1]

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1. Introduction

It is trivial to note that the future is uncertain. It is, however, far from trivial to analyze that uncertainty. The environmental field, in particular, is permeated by uncertainty. Besides the usual economic uncertainties, we have major uncertainties characterizing our knowledge of environmental processes. Often, we simply do not know the long run consequences of interventions in the environment. For example, for many new chemicals, we do not know whether they are carcinogenic or not. Our models of ecosystems dynamics are far from precise. Moreover, future preferences for environmental services are uncertain, which means that future benefits from nature preservation today are uncertain. These topics will be addressed in this chapter. In the next section, we will look at an essentially static framework to look at the role of risk aversion in valuing uncertain environmental benefits. The main tool is the use of quadratic approximations of the von Neuman - Morgenstern utility functions, and the main result is that the benefits from environmental policy reforms depend on risk aversion as measured by the Arrow-Pratt measure of absolute risk aversion and on the variance and covariance of the distributions of preferences and the supply of environmental quality and the wealth (or income) of the individuals. When aggregating the benefits over the whole population of households in the economy, some risks will be highly correlated and it is therefore impossible to bring down the cost of risk bearing by pooling risks. On the other hand, it will of course be possible to reduce the cost of risk bearing by diversification (something which is not studied in this chapter). The key issue of whether the difference between total and expected benefits (called option value in the earlier literature[2]) is positive or negative can be much better understood from the point of view of the covariances between environmental uncertainty and preference uncertainty.

The theory of assets with uncertain returns is very well developed in the corporate finance literature. In particular, the capital budgeting decision is from a formal point of view quite similar to the analysis offered here. However, the main problem we face is that for many natural resource assets, there do not exist markets for pooling and sharing risk. Therefore, no models similar to the CAPM can be developed for those assets. However, much of the analytical framework from that literature can be used for an analysis of decision making on the use of environmental resources when the consequences are risky.

The third part of the chapter looks at the case where information on the state of the world is coming forward with time. Thus, it may be socially profitable to postpone a decision until we know more about the costs and benefits. What's crucial here is that the consequences of a decision, say in the first period of a two-period problem, are difficult or impossible to reverse. There are many examples of this type of problem in the environmental literature, ranging from species loss to climate change. In our analysis of this case, we develop an alternative, dynamic option value concept, called quasi-option

[2] Krutilla et al (1972)

value in the initial contribution by Arrow and Fisher (1974); see also Henry (1974a, 1974b) and Hanemann (1989). Although results here are based on some fairly strong assumptions about the shape of benefit functions and other aspects of the problem, the assumptions are not implausible, at least to a first approximation, in many real-world applications, as we show with an example: whether to develop portions of a tract of land in Central Thailand that is currently part of a national park (Albers, Fisher, and Hanemann, 1996). The rest of the section is devoted to exploring the consequences of relaxing the assumptions, drawing initially on the analytical framework and results of Epstein (1980) and Freixas and Laffont (1984). An interesting question, to our knowledge not treated in the literature, is whether or how it is possible to integrate the two concepts of option value: that arising from the temporal resolution of uncertainty, as in this section, and that based on risk preferences in a static setting, developed in the preceding section. We close with a discussion of this question.

The fourth and final section of the chapter presents a somewhat different approach to the general theory of intertemporal decisions under uncertainty, following closely the treatment in Dixit and Pindyck (1994) and based on the theory of stochastic processes and stochastic calculus. Here we develop first a multi-period model, then continuous time, with special reference to the case of optimal stopping. The latter is important both because a number of environmental and resource problems are appropriately modeled in this way, and also because it is relatively amenable to solution. We illustrate with another example drawn from the literature: the problem of when, if ever, to harvest a stand of old-growth redwood in Northern California that also yields amenity values if left unharvested (Conrad, 1997).

2. Decision Making and Risk: A Static Framework

2.1 The basic framework

As is traditional, we assume that uncertainty can be described by a set of events or states of the world. Each event contains all the information relevant for decision making and the uncertainty consists of not knowing which event that will occur. The particular features characterizing an event we are interested in are income W, supply of environmental resources Q and preferences as measured by utility functions U(W, Q). Note that this means that the individual is not certain what his preferences will be, unless he knows the event that will occur. We will in this section mainly use the indirect utility function, and moreover, we will not, except in the last section consider price uncertainty. One case in which price uncertainty may be quite important is the uncertainty about future interest rates. We will therefore come back to an analysis of that case later. Finally, we will assume that Q is one-dimensional. Generalizations to the case when Q has many but finite dimensions are straight forward.

Let us start by considering one arbitrary individual. For him each event i will describe his income W_i, the supply of environmental services Q_i, and utility function U_i. Let there be a probability measure λ_i over the set of events. The preferences of the individual can then be represented by a von Neuman Morgenstern expected utility.

(2.1.1) $$\int U^i(W_i,Q_i)d\lambda_i = EU^i(W_i,Q_i).$$

This means that we simultaneously will study both what has been called supply uncertainty, that is the uncertainty about Q, and demand uncertainty, i e uncertainty about the preferences. As usual, we will assume that the individual is risk averse. This is in the simplest case defined as the case when the individual refuses to accept a fair bet, i.e. a bet with expectation zero. It is then easy to prove that risk aversion is equivalent to a concave utility functions. However, in our case the utility functions U_i vary from event to event and the situation is slightly more complicated. In view of the confusion about the appropriate definition of risk aversion in this situation, it may be worth while to give a brief analysis of the problem.

2.2 Risk Aversion

In discussing risk aversion, the focus is on the income or wealth variable, so we assume *for this discussion only*, that the environmental quality variable is constant over all possible events. Then we can as well for simplicity suppress that variable. Assume now the individual has to choose between accepting a fair lottery Y with the price Y_i if event i occurs. The conventional definition of risk aversion is now that if E(Y) $\leq$ 0, i.e. if the expected value of the lottery is not positive, then a risk avert individual would not accept a lottery ticket. Based on this definition Schmalensee found that risk aversion means that the marginal utility of income must be constant over all events. However, in view of the fact that income W varies over events, a risk averter may very well accept a lottery with negative expected value if the lottery is negatively correlated with income. In this case will the lottery act as an insurance. Thus we should look at the total lottery W + Y = Z. Assume then that

(2.2.1) $$E(Z) = \overline{Z},$$

if

(2.2.2) $$E(U(Z)) \leq E(U(Z)),$$

then we may, for the moment, say that the individual is risk averse (note that the right hand side is different from U($\overline{Z}$), since U is dependent on which event that will occur). Obviously, if the utility functions are independent of states, then risk version would be equivalent to a concave utility function. However, with state dependent utility functions this is no longer so. The reason is that a positive or negative correlation between Z and the marginal utility of income now becomes an important factor. With positive correlation it may happen that

(2.2.3) $$E(U(Z)) > EU(\overline{Z})).$$

However, this possibility of correlation between marginal utility of income and income has hardly anything to do with common sense interpretation of risk aversion. Therefore, our definition of risk aversion is modified as follows.

Definition *An individual is risk averse if*

(2.2.4) $$E(U(Z) - U(\overline{Z})) \leq 0$$

for all distributions of Z and U provided that they are independently distributed.

It is now easy to see that the state contingent utility function of a risk averse individual must be concave in income and conversely, concave state contingent utility functions imply risk aversion.

In fact, with U and Z independently distributed, the definition can be written as a repeated expectation

(2.2.5) $$E_U E_Z(U(Z) - U(\overline{Z})) \leq 0$$

for all distributions of Z.

A necessary and sufficient condition for this is that for all states

(2.2.6) $$E_Z(U(Z) - U(\overline{Z})) \leq Z\ .$$

Thus, we have the following theorem:

Theorem *An individual is risk averse if and only if his utility function is concave in all events.*

In what follows, we will generally assume that risk aversion characterizes the behavior of the individual and thus we will be using concave utility functions.[3]
We will in the next sections use the expected utility representation of preferences developed above in analyzing some environmental quality decision situation.

2.3 The value of changes in risk

[3]Note that is the same result as Bohm (1975) claimed, although the motivation is different.

Suppose that the different states are characterized by

- i) utility functions U^i
- ii) wealth W_i
- iii) environmental quality Q_i

Moreover, there is a probability measure λ_i over the different states. Then, as we have seen, preferences can be represented by the expected utility

(2.3.1) $$E_\lambda U^i(W_i, Q_i),$$

where E_λ indicates for which distribution the expectation is computed. Let us now consider the case when it is possible to change the probability distribution to λ ' by f .e. environmental policy. Then expected utility is

(2.3.2) $$E_{\lambda'} U^i(W_i, Q_i).$$

Different welfare measures for the change in probability distribution can now be defined. The compensating variation CV and the equivalent variation EV are defined by

(2.3.3) $$E_\lambda U^i(W_i, Q_i) = E_{\lambda'} U^i(W_i - CV, Q_i)$$

(2.3.4) $$E_\lambda U^i(W_i + EV, Q_i) = E_{\lambda'} U^i(W_i, Q_i).$$

These measures have the usual interpretations. CV is the amount that can be taken away from the individual when the probability distribution changes. If CV > 0, then the change in the distribution has increased the expected utility. EV is the amount that would increase expected utility with the same amount as would the change in the probability distribution. Both EV and CV are correct representations of the underlying preference structure and the choice between them is mainly a matter of convenience. Another alternative representation of the environmental change is possible, which implies the one just given, but is more convenient in some applications. Instead of representing the environmental change as a change in the probability distribution, one can look at it as a change in the characterization of each state. Assume then that each state is characterized by environmental quality Q_i so that expected utility is

(2.3.5) $$\int U^i(W_i, Q_i) d\lambda_i.$$

Assume now that the environmental quality changes in each state by ΔQ_i so that

(2.3.6) $$Q_i' = Q_i + \Delta Q_i.$$

The expected utility is now

(2.3.7) $$\int U^i(W_i, Q_i')d\lambda_i .$$

It can be proved (see appendix 1) that provided certain conditions hold, there is a probability distribution λ ' so that

(2.3.8) $$\int U^i(W_i, Q_i + \Delta Q_i)d\lambda'_i = \int U^i(W_i, Q_i)d\lambda_i .$$

With this new representation CV and EV are defined by

(2.3.9) $$E_\lambda U^i(W_i, Q_i) = E_\lambda U^i(W_i - CV, Q_i'),$$

and

(2.3.10) $$E_\lambda U^i(W_i + EV, Q_i) = E_\lambda U(W, Q_i') .$$

The interpretation is exactly the same as the one given above. We will in the sequel only study CV. Define the compensating variation CV_i contingent upon the occurrence of state i as

$$U^i(W_i, Q_i) = U^i(W_i - C_i . Q_i') .$$

We will now try to relate CV to the expected value of CV $_i$. We have from the definitions

(2.3.11) $$E_\lambda U^i(W_i - CV, Q_i') = E_\lambda U^i(W_i, Q_i) = E_\lambda U^i(W_i - CV_i, Q_i') .$$

By making a quadratic expansion around $\overline{W} - CV, and \quad Q'$, where $W = E_\lambda W_i$ and subscripts denote partial derivatives , we have

(2.3.12) $$\begin{aligned} &E_\lambda \left\{ U^i + (W_i - \overline{W})U_W^i + \frac{1}{2}(W_i - \overline{W})^2 U_{WW}^i \right\} = \\ &E_\lambda \left\{ U^i + (W_i - \overline{W} - CV_i + CV)U_W^i + \frac{1}{2}(W_i - \overline{W} - CV_i + CV)^2 U_{WW}^i \right\}. \end{aligned}$$

Define

- $\overline{CV} = E_\lambda CV_i$
- $\text{var}(CV_i) = E_\lambda (CV_i - \overline{CV})^2$

- $\overline{U}_W = E_\lambda U_W^i(\overline{W} - CV, Q_i^{'})$
- $\text{cov}(CV_i, U_W^i) = E_\lambda \left\{ (CV_i - \overline{CV})(U_W^i - \overline{U_W}) \right\}$
- $\text{cov}(CV_i, W_i) = E_\lambda \left\{ (CV_i - \overline{CV})(W_i - \overline{W}) \right\}$

Let us for simplicity assume that $(CV_i - \overline{CV})$ is so small that $(CV_i - \overline{CV})^2$ can be neglected in expression containing $(CV_i - \overline{CV})$, (otherwise we have to solve quadratic equation yielding complicated messy formulas but essentially the same qualitative conclusions). Then we obtain the desired relation between the expected state contingent compensated variations and the compensated variation CV:

(2.3.13)

$$CV = \overline{CV} + \frac{cov(CV_i, U_W^i)}{\overline{U_W}} - \frac{1}{2} E_\lambda \left\{ \frac{U_{WW}^i}{\overline{U}_W} \left[(CV_i - \overline{CV})^2 + 2(CV_i - \overline{CV})(W_i - \overline{W}) \right] \right\}.$$

If we can assume that U_{WW} is state independent, the factor - $\frac{U_{WW}^i}{\overline{U}_W}$ can be identified as the Arrow-Pratt measure R of absolute risk aversion. Then

(2.3.14) $$CV = \overline{CV} + \frac{cov(CV_i, U_W^i)}{\overline{U}_W} + \frac{1}{2} R \{ var(CV_i) + 2cov(CV_i, W_i) \}.$$

This is our basic expression for the value of the change in environmental quality. It is based on a set of not too restrictive assumptions.

2.4 Aggregation over individuals

In general, we are not interested, however, in the individual compensating variation but in the aggregate over the relevant population[4]. Let the population be represented by the set H and denote variables for the individual with a superscript h.

It may happen that the individual state contingent values CV $_i^h$ are dependent on the size of the population. If, for example, the total benefits are independent of the population size, then the individual benefits will in average decrease with increasing size. Even if the benefits are of a public goods nature, congestion may decrease the individual benefits if the number of users increases. Only when we have a pure public good without congestion will the individual compensating variations be independent of the size of population sharing the benefits.

Let therefore H be the measure of the set H (this double use of the letter H will not cause any confusion) and assume that CV $_i^h$ is a function of H CV $_i^h$ (H) In general

[4] We will not introduce a full fledge social welfare functione as it would increase the complexity of the following formulas

we would expect this to be a decreasing function, but with positive externalities it may in fact be increasing.

The total compensating variation is now (CV will now denote the total compensating variation over the whole population and similarly for $\overline{CV}^h \, and \, \overline{CV}$).

(2.4.1)
$$CV = \int_H CV^h dh = \int_H \overline{CV}^h dh + \int_H \frac{cov(CV_i^h, U_W^{ih})}{\overline{U}_W^h} dh + \\ + \frac{1}{2}\int_H R\left\{ var(CV_i^h) + 2cov(CV_i^h, W_i^h) \right\} dh.$$

Let us investigate the three terms in this expression. The first is

(2.4.2)
$$\int_H \overline{CV}^h dh = \overline{CV} \,,$$

which obviously is the aggregated expected state contingent benefits. The second term is

(2.4.3)
$$\int_H \frac{cov(CV_i^h, U_W^{ih})}{\overline{U}_W^h} dh \,.$$

The reason for this term is of course that the marginal utility of income is contingent on the state, and the utility value of the monetary benefits CV $_i^h$ depends on the marginal utility of income. If CV $_i^h$ and U $_i^h$ are independently distributed for each individual h, then this term will vanish. We will in the next section see some examples where such an independence assumption may be reasonable.

Let

(2.4.4)
$$v^{ih} = \frac{U^{hi}}{\overline{U}_W^h}$$

(2.4.5)
$$\overline{CV}_i = \int_H CV_i^h d$$

(2.4.6)
$$\overline{v}^i = \int_H v^{ih} dh \,,$$

and note that $\int v^{ih} d\lambda_i = 1.$

The second term can now be written

$$\int_H \frac{cov(CV_i^h, U_W^{ih})}{\overline{U}_W^h} dh = \int_H \int (CV_i^h - \overline{CV}^h)(v^{ih} - 1) d\lambda_i dh =$$

(2.4.7)
$$= \int \int CV_i^h v^{ih} d\lambda_i dh - H\overline{CV} =$$
$$= \int \int (CV_i^h - \overline{CV}^h)(v^{ih} - \overline{v}^i) d\lambda_i dh + \int \overline{CV}_i (\overline{v}^i - 1) d\lambda_i .$$

The mean value theorem gives an i' such that the last term becomes

(2.4.8)
$$\overline{CV}_i (\overline{v}^{i'} - 1) .$$

If $\overline{CV}_i$ and $\overline{v}_i$ are distributed sufficiently symmetrically,

(2.4.9)
$$\overline{v}^{i'} \sim 1 ,$$

and the expression in 2.4.7 becomes

(2.4.10)
$$H \ cov(CV_i, v^i) .$$

This covariance term characterizes for each state the covariance between the benefits and the marginal utility of income over different individuals. If it can be assumed that different individuals are independent of each other in this respect, the covariance becomes zero and the term

(2.4.11)
$$\int \int cov(CV_i^h v^{ih}) d\lambda_i dh$$

vanishes although the covariance for each individual may be different from zero.
On the other hand, if there is a nonzero covariance over individuals, the term cannot be neglected. This may happen if the variations in benefits and marginal utility of income have a common cause, for example random changes in a certain price. We will look into this later.

There remains the third term,

(2.4.12)
$$\frac{1}{2} \int R^h (var(CV_i^h) + 2cov(CV_i^h, W_i^h)) dh .$$

Obviously, this term represents the cost of bearing the risk of variations in CV_i^h . The following factors influence the size and sign of this term, the covariance between CV_i^h and W_i^h and the variance of CV_i^h , the degree of risk aversion and the joint distribution of these variables over states and individuals.

The next section will be devoted to a discussion of the cost of risk bearing and mechanisms for risk sharing.

2.5 The cost of risk bearing

Assume that utility functions are identical in all states. As in the previous section, the cost for individual h of bearing the risk is

(2.5.1) $$-\frac{1}{2}\frac{U^h_{WW}}{\overline{U}^h_W}\left\{var(CV_i^h)+2cov(CV_i^h,W_i^h)\right\}.$$

When will this cost be positive, negative or zero?

i) $R^h = -\frac{U^h_{WW}}{\overline{U}^h_W} = 0$, that if the individual is risk neutral, the cost of risk bearing is zero. However, we will assume this is not the case, that is $R^h > 0$.

ii) If the bracket $var(CV_i^h)+2cov(CV_i^h,W_i^h)=0$ then the cost is also zero. This can happen if the environmental change generates benefits that are essentially independent of income. If the bracket is negative, then the environmental change will act as an insurance and CV ^{h} $<$ $\overline{CV}^h$ and vice versa if the bracket is positive.

Let us now go back to the aggregate cost of risk bearing and consider different mechanisms for allocating the risk.

A. One mechanism would be to have no risk sharing at all. In that case the social cost of bearing the risk is

$$\frac{1}{2}\int_H R^h\left\{var\quad CV_i^h+2cov(CV_i^h,W_i^h)\right\}dh\,.$$

If all individuals have the same absolute risk aversion and if the joint distribution of benefits and wealth is the product of distributions over individuals and states resp., i.e. if for all h CV $_i^h$ and W $_i^h$ are independent random variables, the aggregate cost is

$$\frac{1}{2}R\left\{\overline{CV}_i+2cov\int_H cov(CV_i^h,W_i^h)dh\right\}.$$

Depending on the distribution of covariance over the individuals, this aggregate cost may be positive, zero or negative.

B. If the risks are "individual risks" in Malinvaud's meaning[5], the aggregate cost above will be approximately zero because the covariance term vanishes. Thus, society should behave in a risk neutral manner, and could achieve that by implementing an

[5] Malinvaud (1972)

insurance scheme. An insurance scheme which is actuarially fair can be described as a measurable function x ^{h} such that E $_\lambda$ x ^{h} = 0. If each consumer chooses the insurance that is best for her, we will have as a result that $\partial U_i / \partial W^h$ is equal across all states. The gain from the optimal insurance x $\overline{x}^h$ for individual h is given by

$$E_\lambda U^h(W^h + \overline{x}^h, Q') - E_\lambda U^h(W^h, Q') \approx \frac{1}{2} R^h var(x^h) \overline{U}_W^h .$$

Thus, the variance term in the expression for CV corresponds to the premium an individual is willing to pay for an insurance that will eliminate the uncertainty. However, such a complete insurance system seems unrealistic in view of the serious problems of moral hazard due to the stochastic nature of preferences. Only if the uncertainty of the preferences is due to an objectively measurable variable can the moral hazard problem be overcome. We will in the next section find a case where such an insurance scheme may exist.

C. Let us now assume that individual benefits are highly correlated. Suppose for simplicity that the set H is finite H = $\{1, 2, \ldots, H\}$ and that individual benefits CV $_i^h$ are constrained by

$$\sum_{h=1}^{H} CV_i^h = Y_i(H),$$

where Y $_i(H)$ is the total benefits in state i. We will see that the assumption that Y $_i$ depends on H is crucial for the results we will derive. Suppose total welfare can be written[6]

$$\int \sum_{h=1}^{H} \beta^h U^h(W_i^h + CV_i^h, Q_i) d\lambda_i .$$

Assuming that lump sum transfers are feasible in each state, the optimal allocation of the total benefits in each state among the individuals is given by

$$\max \sum_{h=1}^{H} \beta^h U^h(W_i^h + CV_i^h, Q_i),$$

s.t.

$$\sum_{h=1}^{H} CV_i^h = Y_i(H).$$

[6]This is not necessary for the analysis. Any assumption that keeps the individual small compared to the total income would give the same result, f. e. if the total benefits are distributed so that $CV_i^h = \beta^h CV_i$ would yield the same result.

The necessary conditions are

$$\beta^h U_{W_i}^h - \mu = 0,$$

where μ is a Lagrange multiplier. The maximum value of the objective function is denoted $V(Y_i(H), Q_i)$. Note that V will not directly depend on the state. The maximum of the objective functions in the original problem can now be written

$$\max \int V(Y_i(H), Q_i) d\lambda_i .$$

Let the Arrow-Pratt measure of absolute risk aversion R_v be defined as

$$R_V = -\frac{V''}{V'}.$$

We know that $V' = \mu$ for all Y and thus,

$$V'' = \frac{d\mu}{dY}.$$

Differentiation of $\beta^h U_W^h = 0$ yields

$$V'' = \frac{U_W^h}{\sum_{h=1}^H \frac{1}{\beta^h}}.$$

Thus,

$$R_V = -\frac{U_{WW}^h}{\beta^h \sum_j \frac{1}{\beta^j} U_W^j}.$$

Let $\frac{1}{\beta^k} U_W^k = \min_j \frac{1}{\beta^j} U_W^j$. Then

(2.5.2) $$R_V \leq -\frac{U_{WW}^h}{\beta^h H \frac{1}{\beta^k} U_W^k} = \frac{1}{H} \frac{\beta^k}{\beta^h} \left[-\frac{U_{WW}^h}{U_W^k} \right] \text{ for all h}.$$

Choose h = k and denote that the individual measure of absolute risk aversion is R_U

(2.5.3) $$R_U = -\frac{U^k{}_{WW}}{U^k{}_W},$$

and so

(2.5.4) $$R_V \leq \frac{1}{H} \frac{1}{R_U}.$$

With increasing size of the population the Arrow-Pratt measure of social risk aversion will therefore go to zero.

The social cost of risk bearing is now

(2.5.5) $$\frac{1}{2}\frac{1}{H}\left\{\operatorname{var}(Y_i(H)) + 2\operatorname{cov}(Y_i(H), W_i(H))\right\},$$

i) If the environmental asset is a pure public good without congestion

(2.5.6) $$Y_i(H) = HY_i,$$

and the social cost of risk bearing is

(2.5.7) $$\frac{1}{2}R_U H\left\{(var(Y_i) + 2cov(Y_i, W_i)\right\}.$$

In this case, an increase in population will increase the total cost of risk bearing[7].

ii) If the asset generates purely private benefits

$$Y_i(H) = Y_i,$$

the cost of risk bearing is

(2.5.8) $$\frac{1}{2}\frac{1}{H}R_U\left\{var(Y_i) + 2cov(Y_i, W_i)\right\}.$$

First, if Y_i and W_i are independent, it follows that the cost of risk tends to zero when the number of individuals sharing the risk increases[8]. If $W_i(H)$ is independent of H, then the cost of risk bearing also tends to zero with the number of individuals sharing the risk. If there are decreasing marginal returns with respect to H, so that $W_i(H)$ is decreasing, the same result obtains.

iii) In the general case, if both

$$\frac{Y_i(H)}{\sqrt{H}} \quad and \quad \frac{W_i(H)}{\sqrt{H}}$$

go to zero with increasing H, the cost of risk bearing will be smaller the larger the population is. The importance of this is obvious. Even if insurance markets cannot work

[7] This is due to A. Fisher (1973)

[8] This result was derived in Arrow and Lind (1972)

because of the correlation of the risks individuals are bearing, it is thus possible to reduce that cost in certain cases by letting more people bear the risk.

2.6 Option prices and option values

Let us apply the theory developed in the last sections to the valuation of a natural asset. Consider the example provided by Schmalensee (1972), i.e. the possible development of Yellowstone National Park which would irreversibly destroy its unique features. The environmental variable Q can in this case assume two values, Q' corresponding to preservation of the National Park and Q" corresponding to irreversible destruction. Uncertainty comes partly from income uncertainty $\left\{W_i^h\right\}$ and partly from preference or utility uncertainty $\left\{U_i^h\right\}$.

CV^h is defined from

$$(2.6.1) \qquad EU_i^h(W_i^h - CV^h, Q') = EU_i^h(W_i^h, Q'').$$

CV^h is known as the option price, i.e. the price the individual is willing to pay for keeping the option of going to Yellowstone in the future[9]. The state contingent benefits from preserving the option, CV_i^h, is defined from

$$(2.6.2) \qquad U_i^h(W_i^h - CV_i^h, Q') = U_i^h(W_i^h, Q'').$$

From the previous sections we know that

$$(2.6.3) \qquad \begin{aligned} CV^h &= \overline{CV}^h + \frac{cov(CV_i^h, U_{i,W}^h)}{\overline{U}_W^h} + \\ &+ \frac{1}{2} R^h \left\{ var(CV_i^h) + 2cov(CV_i^h, W_i^h) \right\}. \end{aligned}$$

The difference between CV^h and $\overline{CV}^h$ is known as the option value OV^h.

$$(2.6.4) \qquad OV^h = CV^h - \overline{CV}^h$$

and a substantial discussion has taken place in the literature whether the option value is positive or negative[10]. It is clear from the formula above that the option value may be of

[9] There are two different uses of the terms option price and option value in the literature. The one used in this section goes back to Weisbrod (1964) and deals basically with cost of riskbearing. The second interpretation is what we are going to study in section 3 and corresponds to the value of information. This second notion was originally called quasi option price and quasi option value.
[10] See Bohm 1975, Schmalensee 1972, Krutilla et al 1972

either sign. However, it is at least possible to outline the factors influencing the size and sign of the option value.
The first term

$$\text{(2.6.5)} \qquad \frac{cov(CV_i^h, U_{i,W}^h)}{\overline{U}_W^h}$$

reflects the collinearity between CV_i^h and U_i^h. If the natural asset is not considered of high importance by the individual (i.e. it not would occupy a big share of her budget if she would have to pay for it), it is hard to see why random variations in the marginal utility of income should be of importance. Thus, this term should be small. Moreover, as we saw in the previous section, if we aggregate over individuals and if CV_i^h, and $U_{i,W}^h$ are distributed independently for each state i over individuals, the aggregate will be close to zero. Thus, there seems to be reasons to assume that this term is negligible. The last term

$$\frac{1}{2}R^h \left\{ var(CV_i^h) + 2cov(CV_i^h, W_i^h) \right\}$$

is on the other hand, more interesting. Obviously, it is impossible to say anything in general about the sign and size of this term. However, for some particular cases, some conclusions may be drawn.

i) If the uncertainty about future preferences is in a sense genuine, knowledge about future income would not increase our ability to predict future benefits. Thus CV_i^h and W_i^h will not be correlated for any individual and the term simplifies to

$$\frac{1}{2}R^h \quad var(CV_i^h),$$

which obviously is positive. In this case risk aversion will imply a positive option value. This is probably the case that corresponds most closely to the problem discussed by Weisbrod (1964) and Cicchetti-Freeman (1971).

ii) The uncertainty about future preferences may be due to uncertainty about some economic variable not explicitly in the utility function. If the natural asset is a recreational facility and if high future oil prices shifts recreational demand from foreign facilities to the domestic asset and vice versa, it is reasonable to assume that CV_i^h and W_i^h are negatively correlated. Thus, the risk premium term for the individual is in this case

(2.6.6) $$\frac{1}{2}R^h\left\{var(CV_i^h)+2cov(CV_i^h,W_i^h)\right\}$$

may have either sign depending on whether the second term dominates the first or not. It may therefore happen that the individual has a positive option value. However, from the point of view of society, if the recreational asset is such that $\overline{CV}_i$ increases less than in proportion with the size of the population, the term $\frac{1}{H}\overline{CV}$ will be small and the covariance term will dominate.

Thus, even if the individual has a positive option value, society may, in spite of this, have a negative option value. The reason for this is that the risk component corresponding to the variance can be better shared through society and the remaining risk component is essentially an insurance against the future oil prices. In particular, this implies that investments in this facility should be discounted with a negative risk premium. Of course, if the facility is a pure public good with no congestion, then this result does not hold, and the risk term for society is simply the individual risk premium multiplied with the size of the population.
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2.7 Price uncertainty

Let us introduce the interest rate r as a variable in the indirect utility function and let us also assume that r is a stochastic variable. We will also neglect the many consumers case and instead assume that the utility represents social utility. The utility function is now

(2.7.1) $$U_i(W_i,r,Q_i),$$

where i as previously represent the state of the world. We can now go through the same kind of exercise as we have done in previous sections in order to derive approximations to the true CV. The result is under essentially the same assumptions as before

(2.7.2) $$\begin{aligned}CV &= \overline{CV}+\frac{cov(CV_i,U_{i,W})}{\overline{U}_W}+\frac{1}{2}R\{var(CV_i)+2cov(W_i,CV_i)\}-\\&-\frac{U_{r,W}}{\overline{U}_W}cov(r,CV_i),\end{aligned}$$

where $U_{r,W}=\partial^2U/\partial r\partial W$ has been assumed to be state independent. We will assume that $U_{r,W}<0$, that is, an increase in the price r will reduce the marginal utility of wealth. It is interesting to note that even if the individual is risk neutral so that the utility function is linear in wealth, the last term may be different from zero. In the simple case when preferences are state independent and represent risk neutral behavior we would have

(2.7.3) $$CV=\overline{CV}-\frac{U_{r,W}}{\overline{U}_W}cov(r_i,CV_i).$$

Thus, the possible correlation between on the one hand the state contingent benefits CV_i and the price (or the interest rate) will create a difference between the expected benefits $\overline{CV}$ and the true benefits CV. Is such a correlation to be expected? Should not the covariance between state contingent compensation variations and the interest rate be zero in general? The interest rate will either reflect the desired trade off between consumption in different time periods or the future marginal productivity of capital (or in perfectly competitive equilibrium both). In the latter case, the interpretation is that the interest rate gives an indication of the necessary capital investment today in order to increase consumption with one unit in a future time period. But the future marginal productivity of capital will in general be influenced of the availability of natural resources. An increase in the availability of resources will in general increase the expected future productivity and thereby the interest rate. If we are analyzing a project with large environmental consequences, not only will the cost be large and state contingent benefits small because of that, but the interest rate will be small in the states that correspond to small benefits. Thus we should expect a negative covariance between CV and r. This means that, even in the absence of risk aversion and with state independent preferences, will the true benefit be smaller than the expected compensating variation. The covariance between interest rate and the benefits will thus acts as if the decision maker is risk avert. Another way of seeing that is to look at the following formula for the present value of a current project with future environmental consequences.

(2.7.4)
$$NB_0 = C_0 - \frac{C_1}{1+r},$$

where C_O is the present benefit, C_1 is the future cost from environmental degradation, r is the interest rate, and NB_0 is the present value of the net benefits. Assume C_1 and r are stochastic variables and assume that the decision maker is risk neutral. The standard procedure is to calculate the number

(2.7.5)
$$C_0 - \frac{\overline{C}_1}{1+\overline{r}},$$

where $\overline{C}_1$ and $\overline{r}$ are the expected values of C_1 and r. The expected value of NB_0 is, however,

(2.7.6)
$$E(NB_0) = C_0 - E\left\{\frac{C_1}{1+r}\right\} \cong C_0 - E\{C_1(1-r)\} \cong C_0 + \frac{\overline{C}_1}{1+\overline{r}} - cov(r, C_1).$$

Once again, we see that the covariance between r and C_1 will affect the expected net benefits and with a positive correlation between r and C_1, the expected net benefit will be less that what would be calculated with the conventional practice.

3. Intertemporal resolution of uncertainty

3.1. Introduction

We consider the following situation. Assume a project is designed that will include the use of a particular environmental resource over several time periods. However, the value of this resource in future periods is not known with certainty. There is a possibility that information on these values may be generated over time. What is the appropriate decisionmaking framework? In most analyses of uncertainty of future environmental resource use, especially empirical applications, the information structure has been very simple. All information is (assumed to be) contained in an a priori probability distribution with no more coming available at a later stage. For some applications, this may be a realistic assumption. Alternatively, if the decision to use a natural asset in a special way can be completely (and costlessly) reversed at a later stage, the prospect of getting more information in the future does not have to be included in the analysis. In this case there are no essential intertemporal connections. In the other extreme when the decision is completely irreversible, the prospect of better information may be quite important, as we shall show in this section. Before proceeding, we need to consider further the concept of irreversibility in the setting of environmental decisions. There are three issues here: (1) are the consequences of a decision irreversible?; (2) will information about the environmental values be forthcoming in the future?; and (3) how should the prospect of this information be taken into account in a decision to be made in the present, or the first period? We offer a few remarks on the first. On the assumption that the answer to the second may be "yes", the third is the subject of the subsequent analysis in both this section and the next.

As suggested in the Introduction to this Chapter, environmental impacts of an investment in resource development can be long lasting, or even irreversible. This is a feature of environmental valuation and decision problems that has received a great deal of attention in the literature, based on findings in the natural sciences. For example, there is both scientific and popular concern today about loss of biodiversity, the genetic information that is potentially valuable in medicine, agriculture, and other productive activities. Much of the concern is for endangered species, or the habitats such as tropical moist forests that are subject to more or less irreversible conversion to other uses. But even if species survival is not at issue, biological impacts can be very difficult to reverse over any relevant time span. The clear-cutting of a climax forest species, for example, removes the results of an ecological succession that may represent centuries of natural processes. Regeneration may not lead to the original configuration, as opportunistic species such as hardy grasses come in and preempt the niche otherwise filled by the climax species (Albers and Goldbach, 2000).

Irreversibilities have also been identified as a key feature of the problem of how to respond to potential impacts of climate change. Emissions of greenhouse gases, in particular carbon dioxide, accumulate in the atmosphere and decay only slowly.

According to one calculation, assuming business-as-usual use of fossil fuels over the next several decades, after a thousand years carbon dioxide concentrations will still be well over twice the current level, and nearly three times the pre-industrial level – and will remain elevated for many thousands of years (Schultz and Kasting, 1997). There is also some prospect of essentially irreversible catastrophic impact as would result for example from the disintegration of the West Antarctic Ice Sheet and consequent rise in sea level of 15-20 feet. Recent findings suggest that this possibility is more serious, and perhaps closer in time, than economists (and others) have realized (de Angelis and Skvarca, 2003).

On a less grand scale, studies of the dynamics of ecosystems have uncovered positive feedbacks, which give rise to multiple equilibria. Movements from one such equilibrium to another will almost always be characterized by hysteresis and frequently by irreversibility, as illustrated in several cases discussed by Dasgupta and Mäler (2003).

Although we have been speaking of costs and benefits experienced over several periods, the analysis in this section will focus on two-period models. These are sufficient to develop many of the main results on the temporal resolution of uncertainty (though an empirical application we shall discuss is based on a three-period model, a straightforward extension of the two-period model in the next subsection). Section 4 develops more formally the multi-period and continuous-time cases, along with another empirical application based on results there.

3.2 A Simple Two-Period Model

Consider the problem, originally studied by Arrow and Fisher (1974) and Henry (1974a, 1974b), as set out in Fisher and Hanemann (1986), of choosing whether to preserve or develop a tract of land in each of two periods, present and future. The development, we assume, is irreversible. Future benefits of development and preservation are uncertain, but we learn about them with the passage of time. In this simplest case, we assume that the uncertainty about future benefits is resolved at the start of the second period.

Let the benefit from first-period development, net of environmental costs (the benefits of preservation), be $B_1(d_1)$, where d_1, the level of development in period 1, can be zero or one. The present value of the benefit from second-period development is $B_2(d_1 + d_2, \theta)$, where d_2 can be zero or one and θ is a random variable. Note that, if $d_1 = 1$, $d_2 = 0$.

We want the first-period decision to be consistent with maximization of expected benefits over both periods. If benefits are measured in utility units, then this is equivalent to expected-utility maximization. But we can allow benefits to be measured in money units so that the results we shall obtain do not depend on risk aversion.

Let $\hat{V}(d_1)$ be the expected value over both periods as a function of the choice of first-period development ($d_1 = 0$ or $d_1 = 1$) given that d_2 is chosen to maximize benefits in the second period. Then, we have, for $d_1 = 0$, (3.2.1) $$\hat{V}(0) = B_1(0) + E\left[\max_{d_2}\left\{B_2(0,\theta), B_2(1,\theta)\right\}\right].$$

Second-period development, d_2, is chosen at the start of the second period when we learn whether or not $d_2 = 0$ or $d_2 = 1$ yields greater benefits. At the start of the first period, when d_1 must be chosen, we have only an expectation, $E[\cdot]$, of the maximum.

If $d_1 = 1$, we have

(3.2.2) $$\hat{V}(1) = B_1(1) + E\left[B_2(1,\theta)\right].$$

With development in the first period, we are locked into development in the second ($d_1 = 1 \Rightarrow (d_1 + d_2) = 1$). To get the decision rule for the first period, $\hat{d}_1$, compare:

(3.2.3) $$\hat{V}(0) - \hat{V}(1) = B_1(0) - B_1(1) + E\left[\max_{d_2}\left\{B_2(0,\theta), B_2(1,\theta)\right\}\right] - E\left[B_2(1,\theta)\right],$$

and choose

(3.2.4) $$\hat{d}_1 = \begin{cases} 0 \; if \; \hat{V}(0) - \hat{V}(1) \geq 0 \\ 1 \; if \; \hat{V}(0) - \hat{V}(1) < 0 \end{cases}.$$

Now, let us suppose that, instead of waiting for the resolution of uncertainty about future benefits before choosing d_2, we simply replace the uncertain future benefits by their expected value. This would be appropriate if we did not expect to receive information, over the first period, that would permit us to resolve the uncertainty. In this case, the expected value over both periods, for $d_1 = 0$, is

(3.2.5) $$V^*(0) = B_1(0) + \max_{d_2}\left\{E\left[B_2(0,\theta)\right], E\left[B_2(1,\theta)\right]\right\}.$$

Second-period development, d_2, is in effect chosen in the first period, to maximize expected benefits in the second period, because we do not assume that further information about second-period benefits will be forthcoming before the start of the second period. For $d_1 = 1$,

(3.2.6) $$V^*(1) = B_1(1) + E\left[B_2(1,\theta)\right].$$

As before, development in the first period locks in development in the second. Comparing (3.2.5) and (3.2.6),

$$V^*(0) - V^*(1) = B_1(0) - B_1(1) + \max_{d_2}\left\{E\left[B_2(0,\theta)\right], E\left[B_2(1,\theta)\right]\right\} - E\left[B_2(1,\theta)\right], \tag{3.2.7}$$

and

$$d_1^* = \begin{cases} 0 \text{ if } V^*(0) - V^*(1) \geq 0 \\ 1 \text{ if } V^*(0) - V^*(1) < 0. \end{cases} \tag{3.2.8}$$

How do the decision rules in (3.2.4) and (3.2.8) compare? First, notice that

$$\left[\hat{V}(0) - \hat{V}(1)\right] - \left[V^*(0) - V^*(1)\right] = \hat{V}(0) - V^*(0), \tag{3.2.9}$$

since $\hat{V}(1) = V^*(1)$. Then,

(3.2.10)

$$\hat{V}(0) - V^*(0) = E\left[\max_{d_2}\left\{B_2(0,\theta), B_2(1,\theta)\right\}\right] - \max_{d_2}\left\{E\left[B_2(0,\theta)\right], E\left[B_2(1,\theta)\right]\right\}.$$

Finally,

$$\hat{V}(0) - V^*(0) \geq 0, \tag{3.2.11}$$

from the convexity of the maximum function and Jensen's Inequality, which states that the expected value of a convex function of a random variable is greater than or equal to the convex function of the expected value of the random variable.[i]

It is this difference, $\hat{V}(0) - V^*(0)$, that has been called option value in some of the environmental literature, or as in Arrow and Fisher (1974), quasi-option value. Note that option value in this formulation cannot be negative. It may also be interpreted as a (conditional) value of information: the value of information about future benefits *conditional* on retaining the option to preserve or develop in the future ($d_1 = 0$).[ii]

Another important result is that, from equations (3.2.4), (3.2.8) , and (3.2.11) , we see that there will be cases in which development is optimally undertaken in the first period in the no-information ("star") scenario, but not in the new-information ("hat") scenario. However, the reverse cannot occur, that is, there will be no cases in which first period development is optimal with the prospect of future information, but not with the prospect of no information. In other words, current development is less likely with the prospect that information about uncertain environmental values will be forthcoming in the future. In the next section this result, along with the potential significance of option value, is illustrated in an empirical application to the choice of development alternatives in a forested region of Thailand.

3.3 An Empirical Application: To Develop or to Preserve

Most applied benefit-cost analyses of environmental decision problems of course employ the "open-loop" formulation in which random variables are simply replaced by their expected values, the "star" scenario considered above. More sophisticated decision analyses, based on a "closed-loop" approach, in which the agent takes into account the prospect of new information, are however beginning to emerge in the literature (the "hat" scenario). An example is the analysis of implications of uncertainty and irreversibility for valuation and management of a tract of tropical forest land in Thailand, by Albers, Fisher, and Hanemann (1996). The problem is specified as one of allocating the forest among three competing uses, P (for preservation), M (for intermediate uses), and D (for development), over three periods, to maximize the expected benefits of use. The pattern of feasible sequences of uses is displayed in Figure 1. Note the greater complexity than in the two-use, two-period model, which involves just three feasible sequences: $P \rightarrow P, P \rightarrow D, and\ D \rightarrow D.$ The analytical approach, and results, are however exactly analogous to those obtained in the simpler model.

If no information about future benefits is anticipated, the maximum expected present value associated with putting the forest tract to the preservation use in the first period is

(3.3.1)
$$\begin{aligned} V_P^* = P_0 &+ \max\{E[P_1] + \max\{E[P_2], E[M_2], E[D_2]\}, E[M_1] \\ &+ \max\{E[M_2], E[D_2]\}, E[D_1] + E[D_2]\}, \end{aligned}$$

where $P_i, M_i, and\ D_i$ represent the benefits from preservation, intermediate uses, and development in period i and the expectation is with respect to the information set available in the first period. This expression is analogous to V*(0) in the two-period preservation vs. development problem. If, however, better information about future benefits and costs will be forthcoming, this affects not only future decisions that will be made on the basis of the information, but also the current decision, again as in the two-period problem. Specifically, Albers, Fisher, and Hanemann assume that, at the start of each period, the decision maker learns what the benefits of each of the alternative uses of the tract will be in that period (though not in future periods) and then chooses the highest-yielding alternative. In this formulation, the maximum expected present value associated with preservation in the first period is

(3.3.2)
$$\begin{aligned} \hat{V}_P = P_0 &+ E[\max\{P_1 + \max\{P_2, M_2, D_2\}, M_1 \\ &+ \max\{M_2, D_2\}, D_1 + D_2\}], \end{aligned}$$

and

$$\begin{aligned} \hat{V}_P - V_P^* = E[&\max\{P_1 = \max\{P_2, M_2, D_2,\}, M_1 + \max\{M_2, D_2\}, D_1 + D_2] \\ &- \max\{E[P_1] + \max\{E[P_2], E[M_2], E[D_2]\}, E[M_1] \\ &+ \max\{E[M_2], E[D_2], E[D_1] + E[D_2]\} \\ &\geq 0, \end{aligned} \tag{3.3.3}$$

again, as in the two-period model, from the convexity of the maximum function and application of Jensen's Inequality. The difference, $\hat{V}$ – V*, is option value. As we shall see, it turns out to be small, relative to the total value of the site, in this application, but to have a dramatic impact on optimal use in the first period.

The area studied is partly included in an existing park, Khao Yai National Park (KYNP), in central Thailand. The analysis divides KYNP into four management units or plots. The outer edge of the park, plot 1, has been encroached and begins in the use M. The inner plots, 2 and 3, begin in the preservation use P. The fourth zone, also in use P, is not currently protected by the park system but is under consideration for annexation.

The preservation benefits include erosion control, hydrologic functions, tourism, and extractive goods. Two states of the world define the uncertainty about future preservation benefits. The high state reflects the additional value that a viable population of Asian elephants creates in KYNP. High-state benefits grow at 21/2 percent per year. The low state reflects the possibility of a large drop-off in tourism revenue in the later periods. The high state occurs with a probability of 0.5 in the second period and 0.25 in the third period.

The intermediate uses, which we do not focus on here, include more intensive extractive activities and small scale shifting cultivation. The irreversible development option generates income through permanent agriculture and eucalyptus plantations. The agricultural values decline over time at a rate that corresponds to erosion-induced productivity declines on the fragile tropical soils. Following Tongpan *et al*. (1990), a 12-year investment horizon is assumed for the eucalyptus plantations. That 12-year plan, a function of rotation age, defines the length of the planning periods in the analysis.

The open-loop and closed-loop optimizations preserve markedly different amounts of KYNP. Both convert the degraded plot 1 to the development option due to its relatively small returns to preservation; convert plot 4 to intensive extractive goods and small-scale agriculture, the intermediate use M; and preserve plot 2. They differ, however, on plot 3. The open-loop optimization develops this plot and thereby reduces the amount of local parkland by half. The correct, closed-loop approach preserves this area and maintains the opportunity to take advantage of future information about preservation values.

The option value generated in this example constitutes 1.6 percent of the total value over the three periods in the closed-loop calculation. Despite its small size, this additional value creates a dramatic difference in the optimal land-use pattern. Moreover, there are reasons to believe that the option value in the example is understated. Option value will be larger in situations of uncertainty in near periods and in cases of large divergences in possible outcomes (Fisher and Hanemann, 1986). The latter is probably important in the global valuation of tropical forests, given their potential, though uncertain, significance for biodiversity conservation and global climate control

3.4 Some Extensions of the Arrow-Fisher-Henry-Hanemann Analysis

Now let us take up the agenda mentioned in the Introduction to the Chapter: relaxing implicit or explicit assumptions in a model like that presented in section 3.2. We consider the following extensions:

- sharper definitions of the meaning of forthcoming information
- more general benefit functions
- active search for information instead of passively waiting for it
- change that can be reversed at a cost
- uncertainty about irreversibility
- option value as a risk premium in the sense of section 2, when there are irreversible changes

3.5 Bayesean updating[11]

We will consider the following decision-making situation. We are studying two time periods. In time period one a decision has to be made on variable X $_{X_1}$ and in time period two a decision on X $_2$. X $_1$ and X $_2$ can be thought of resource use. Once development has been made, it is impossible to restore the resource and therefore X $_1 \leq X_2$. The pay-off is given by the expected value of the utility function

$$U(X_1, X_2, Z), \tag{3.5.1}$$

where Z is a random variable.

When the decision is made in the first period, the only information on Z is an a priori probability distribution r. We assume that Z only can take a finite number of values, i. e. $Z=(Z_1, ..., Z_m)$ and the corresponding probabilities are $r = (r1, ... ,r_m)$. After the decision on x_1 has been made, but before the decision on x_2 is made, the decision maker gets a signal Y. Y is a random variable taking the values $(Y_1, ... , Y_n)$ with probabilities $q = (q_1, ..., q_n)$.

If Y and Z are perfectly correlated, the decision maker knows with probability one which realization of Z that will occur, but if Y and Z are independent, the signal Y gives

[11]This section is based on Epstein 1980

no information at all on Z. In the general case, we know that the decision maker will revise his probabilities according to Bayes' theorem[12] . Let be the conditional probability distribution of Z given the signal Y be

(3.5.2.) $$\pi_{i,j} = Pr(Z = Z_i \mid Y = Y_j) = \frac{\Pr(Y = y_j \mid Z = z_i) r_i}{\sum_{k=1}^{m} \Pr(Y = y_j \mid Z = z_k)}.$$

Let

(3.5.3) $$\lambda_{j,i} = Pr(Y = Y_j \mid Z = Z_i)$$

be the likelihoods. Then

(3.5.4) $$\pi_{j,i} = \frac{r_i \lambda_{i,j}}{\sum_k r_k \lambda_{k,j}}.$$

Let $\Lambda = [\lambda_{j,i}]$ and $\Pi = [\pi_{i,j}]$.

Then we also have (for any matrix A, A^T means the transpose of A)

(3.5.5) $$\Pi q = r,$$

and

(3.5.6) $$r^T \Lambda = q^T.$$

3.6 Information structures

The signal Y is given within a given information structure defined by (Y and the probabilities q and Λ . Another information structure, Y', is defined by (Y_1 ', ..., Y'_n), q', and Λ' . Obviously, if we have the same priors r, $\Pi q = \Pi' q'$ = r and $r^T \Lambda' =$ q'. Our decision making problem can now be written

(3.6.1) $$\max_{x_1} \sum_{j=1}^{m} q_j \max_{x_2} \sum_{i=1}^{n} \pi_{i,j} U(X_1, X_2, Z_i),$$

where X_1', and X_1'', must of course be chosen within the feasibility sets. In analyzing this problem we follow and we will introduce a new notation. Let $\xi = [\xi_1, ..., \xi_n]$ such

[12]See [Raiffa1968] for an extremely well-written presentation of Bayesian decision making.

that $\xi_i \geq 0$ and $\sum_{i=1}^{n}\xi_i = 1$. Define

(3.6.2) $$J(X_1,\xi) = \max_{x_2}\sum_i \xi_i U(X_1,X_2,Z_i)\,.$$

J can be interpreted as the maximum expected utility from using X_1 in the first period, given the probability distribution ξ. We can then formulate the decision problem as follows:

(3.6.3) $$\max_{X_1}\sum_j q_j J(X_1,\pi_j)\,,$$

where π is the j:t column of Π . Following Blackwell (1951) and Marshak and Miyasawa (1968), let us define the concept of "more informative". Consider two information schemes Y and Y' corresponding to the same prior probability distribution. The corresponding posterior probabilities are π and π' and the probabilities for the signals are q and q' respectively. Y is defined to be more informative than Y' if and only if

(3.6.4) $$\sum_j q_j J(X_1,\pi_j) \geq \sum_j q_j^{'} J(X_1,\pi_{i,j}^{'})$$

for all X_1, all utility functions U and all feasibility sets. Y more informative than Y' then means that independent of the initial choice of x_1 and the utility functions, Y will give a higher well-being, that is the signal Y enables us to achieve a higher well-being than signal Y'. In order to get a feeling of the meaning of this definition, let us consider two extreme cases:

i) Y' means that $\pi'_{i,j} = r_i$ j = 1, . . , m, that is the signal Y' does not carry any new information.

ii) Y'' implies perfect information, that is m = n and

$$\pi''_{i,j} = \begin{cases} 0 \text{ if i} \neq j \\ 1 \text{ if i=j} \end{cases},$$

so that the signal reveals with certainty which z that will occur. Obviously q = r. If Y is an arbitrary information structure our intuition requires that Y' is more informative than Y which in turn should be more informative than Y'. That this is, indeed, the case is easily proved.
We first have

$$\pi''q'' = \pi g = \pi' q' = r\ .$$

Then, from the definition of J it follows that J is convex in ξ (maximum of a linear function in X). Thus,

$$\sum_j q_j J(x_1,\pi_j) \geq J(x_1, \sum_j q_j \pi_j) = J(x_1, r) = J(x_1, \sum_j q_j' \pi_j') = \sum_j q_j' J(x_1, \pi_j)$$

for all U and X_1. Thus Y is more informative than Y'. Note that in this case, we have

$$max_{x_1} q_j max_{x_2} \sum_i \pi_i ' U(x_1, x_2, z_i) = max_{x_1} \sum_j q_j max \sum_i r_i U(x_1, x_2, z) =$$
$$= max_{x_1, x_2} \sum_i r_i U(x_1, x_2, z_i),$$

that is, we maximize the unconditional expectation of the benefits, and no adjustments need to be done with respect to forthcoming information. We also have

$$\sum_{j=1}^m q''_j \max_{x_2} \sum_i \pi''_{i,j} U(x_1,x_2,z_i) = \sum_j q''_j \max U(x_1,x_2,z_j) =$$
$$\sum_i r_i \max_{x_2} U(x_1,x_2,z_i) = \sum_j q_j \sum_i \pi_{i,j} \max_{x_2} U(x_1,x_2,z_i) \leq$$
$$\sum_j q_j \max_{x_2} \sum_i \pi_{i,j} U(x_1,x_2,z_i)$$

for all U and x1 and for all feasibility sets. Thus Y" is more informative than Y. Another way of understanding the definition of being more informative than is to look at the case when

$$Z = Y + \delta,$$

where δ is the observation error Assume that δ is normally distributed with zero mean and is independent of Z. It is now easy to demonstrate that if we are comparing two distributions of δ, the one with smaller variance is more informative than the other (Bradford and Kelian (1977)) proved the following characterization of Y being more informative than Y'.

Theorem *Y is more informative than Y' if and only if for every convex function ρ on S^{m-1} (S^{m-1} =* $\{\xi \in R^m; \xi_i \geq 0,\ \sum_i \xi_i = 1\}$)

(3.6.5) $$\sum_j q_j \rho(\pi_j) \leq \sum_j q_j' \rho(\pi_j').$$

For a proof see Marshak and Miyasawa (1968).

3.7 Irreversibility

Using the characterization given by Marshak and Miyasawa, Epstein proved the following theorem:

Theorem *Let $J(X_1, \xi)$ be concave in X_1. Assume Y is more informative than Y'. Let X_1^* and X_1^{**} maximize $\sum q_j J(x_1, \pi_j)$ resp. $\sum q'_j J(x_1, \pi_j')$ where (q, π) and (q',π') correspond to information structures Y resp. Y'. If $\partial J / \partial x_1$ is concave (convex) in* ξ, $X_1^* \leq X_1^{**}$ $(X_1^* \leq X_1^{**})$.

Proof

The simple proof goes as follows: Let $J_{x_1} = \partial J / \partial x_1$ and let us assume that J $_{X_1}$ is concave in ξ. Then

$$0 = \sum_j q_j J_{X_1}(x_1^*, \pi_j) = \sum_j q_j' J_{X_1}(x_1^{**}, \pi_j') \leq \sum_j q_j' J_{X_1}(x_1^{**}, \pi_j')$$

because of the assumption that Y is more informative than Y' and the Marshak-Miyasawa theorem . Then, it follows that X Similarly for the case when J $_{X_1}$ is convex in ξ.

Note that convexity and concavity conditions are only sufficient conditions for the result! Furthermore, if π is different from π' and J $_{X_1}$ is strictly concave, the weak inequalities will be replaced by strict inequalities. Using this theorem, Epstein further showed that

Theorem *Assume that X $_1$ can take only the values 0 and 1. Furthermore, assume the constraint X1 (irreversibility). Then, if X $_1^{**} = 0$, it follows that X $_1^{*} = 0$.*

As J is concave (convex) in X $_1$, J $_{X_1}$ is non-increasing (non-decreasing) and X $_1 \leq X_1'$ $(X_1 \geq X_1'$.

It is now possible to use this theorem to derive results on the irreversibility effect. In both cases X $_1'$ and X $_1$" are either 0 or 1. Suppose X $_1$" = 0, and hence

.

$$\sum_{j=1}^{m} q_j\,"J(0, \pi_j\,") > \sum_j q_j\,"J(1, \pi_j\,").$$

As Y' is more informative than Y, we have

$$\sum_{j=1}^{m} q_j' J(0, \pi_j') \geq \sum_j q_j\,"J(0, \pi\,").$$

Moreover, as the change is irreversible, that is x $_2 \geq x_1$,

$$\sum_j^{m} q_j\,"J(1, \pi_{ij}\,") = \sum_j \sum_i q_j\,"\pi_{ij}\,"U(1,1,z_i) =$$
$$= \sum_i r_i U(1,1,z_i) =$$
$$= \sum_j \sum_i q_j' \pi_{i,j}' U(1,1,z_i) = \sum_j q_j' J(1, \pi_i').$$

Combining the above, we have

$$\sum_j q_j' J(0,\pi_j') > \sum_j q_j' J(1,\pi_j'),$$

and $x_1' = 0$.

Note that the inequality $x_1 \leq x_2$ reflects an irreversibility in the decision. If a resource use equal to x_1 has been decided, future resource use must be equal to or exceed this amount. Thus, if we are going to make an irreversible decision (building a hydropower plant, developing Yellowstone National Park to an industrial site etc.), then the prospect of getting more information in the future on costs and benefits will not increase the benefits of undertaking the development now. However, this theorem was based on the assumption that the optimal value of x_1 is either 0 or 1. The theorem is not necessarily true if the assumptions yielding this are abandoned, unless other restrictions are introduced. Such restrictions apply to the net benefit or utility function. Assuming that the U function takes the special form

$$U(X_1, X_2, z_i) = U(X_1) + V(X_2, z_i),$$

where U and V are strict concave functions of X_1 and X_2 a corresponding result can be derived. We are thus considering the problem

$$\max_{X_1}\{U(X_1) + \sum_j \max_{X_2 \geq X_1} \sum_i \pi_{ij} V(X_2, z_i)\}.$$

X_1 may be interpreted as the development in period 1, giving net benefits $U(X_1)$ and X_2 is the total development in the next period yielding the present value of the net benefits equal to $V(X_2, z_i)$ The irreversibility is expressed in the condition $X_2 \geq X_1$.

The J function becomes

$$J(X_1,\xi) = \max_{X_2 \geq X_1} \sum_i \xi_i V(X_2, z_i).$$

Assume that $X_2'(X_1)$ solves this maximum problem, where X_2' is continuous and piecewise differentiable. Then

$$J(X_1,\xi) = \max_{X_2 \geq X_1} \sum_i \xi_i V(X_2'(X_1), z_i)).$$

Furthermore,

$$\frac{dX_2'}{dX_1} = \begin{cases} 0 \text{ if } X_2' > X_1 \\ 1 \text{ if } X_2' = X_1, \end{cases}$$

and therefore,

$$\frac{\partial J}{\partial X_1} = \sum_i \frac{\partial V(X_2', z_i)}{\partial X_{2ii}} \frac{dX_2'}{dX_1} = \begin{cases} 0 \text{ if } X_2' > X_1 \\ \sum_i \xi_i \frac{\partial V}{\partial X_2'} \text{ if } X_2' = X_1, \end{cases}$$

Moreover, $X_2' > X_1$ if and only if

$$\sum_i \xi_i \frac{\partial V(X_2', z_i)}{\partial X_2} > 0\,.$$

Thus,

$$\frac{\partial J}{\partial X_1} = \min\left\{0,\ \sum_i \xi_i \frac{\partial V}{\partial X_2}\right\}.$$

As both terms in the bracket are concave, it follows that $\frac{\partial J}{\partial X_1}$ is concave in ξ. Then it follows from Epstein's theorem that

$$X_1' \leq X_1".$$

We then have the following theorem;

Theorem *If U(X $U(X_1, X_2, z_i) = U(X_1) + V(X_2, z_i)$ and U(X_1) and v(X_2, z_i) are concave functions of X_1, and X_2, respectively, and if the optimal value for X_1 with information structure Y' is X_1', and with information structure Y is X_1", where Y' is more informative than Y", then*

$$X_1' \leq X_1".$$

Thus, a fairly general proposition has been established. If more information will be available in the future, "less" irreversible changes should be undertaken now.

3.8 Irreversibility at a cost

Let us now consider the case when it is possible to restore the development but at a cost. This means that we will replace the restriction $X_2 \geq X_1$ with a cost function for the measures that are necessary to make $X_1 \geq X_2$ feasible. If the decision in the first period is X_1, assume it is possible to restore the resource in the second period to $X_2 < X_1$ but at a cost given by the cost function c(X_1, X_2) defined by

(3.8.1) $$c(X_1, X_2 = \begin{cases} 0 \text{ if } X_1 \leq X_2 \\ \gamma(X_1 - X_2) \text{ if } X_1 > X_2 \end{cases}$$

with γ as a constant. Note that with this cost function, cost is a continuous function of the amount of restoration but the marginal cost is discontinuous at zero restoration. We will later look at a different formulation of the cost function. As in the previous section, we define the J-function (making the same intertemporal separability assumption as before) as

(3.8.2) $$J(X_1, \xi) = \max_{X_2} \left\{ \sum_i \xi_i V(X_2, z_i) - c(X_1, X_2) \right\}.$$

Define $\overline{X}_2$ from

(3.8.3) $$\max_{X_2} \sum_i \xi_i V(X_2, z_i) = \sum_i \xi_i V(\overline{X_2}, z_i),$$

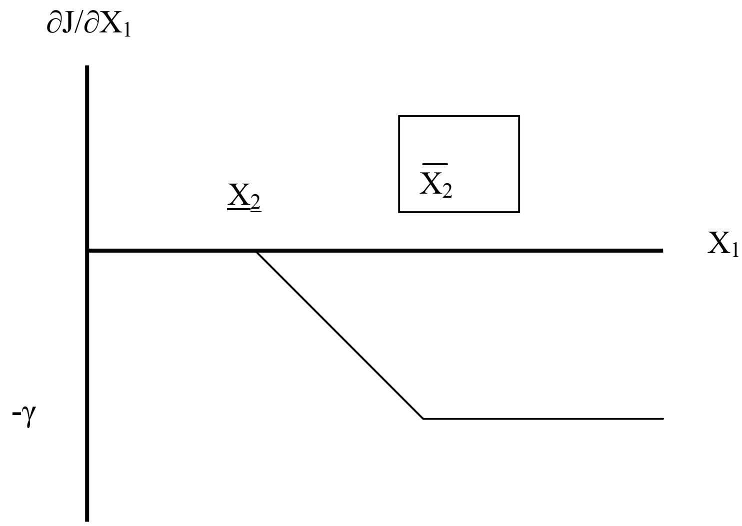
that is, $\overline{X}_2 = \arg\max \sum_i \xi_i V(X_2, z_i)$. Assume V is differentiable and define $\underline{X}_2$ from

(3.8.4) $$\sum_i \xi_i \frac{\partial V(\underline{X}_2, z_i)}{\partial X_2} + \frac{\partial c(X_1, \underline{X}_2)}{\partial X_2} = \sum_i \xi_i \frac{\partial V(\underline{X}_2, z_i)}{\partial X_2} - \gamma = 0\,.$$

$\overline{X_2}$ is the upper bound for the set of X_1 at which no restoration will take place and $\underline{X}_2$ is similarly the lower bound for the set of ´X_1 at which restoration will take place. As V is concave, it follows that $\underline{X}_2 < \overline{X}_2$ Let us now study the choice of optimal X_2 * , contingent upon the choice of X_1 . We have:

i) If 0 $0 \le X_1 \le \overline{X}_2$, then $X_2^*(X_1) = \overline{X}_2$, and $\partial J / \partial X_1 = 0, dX_2^* / dX_1 = 0$,
ii) If $\overline{X}_2 \le X_1 \le \underline{X}_2$, then $X_2^*(X_1) = X_1$ and $\partial J / \partial X_1 = \sum_i \xi_i \frac{\partial V(X_2^*(X_1), z_i)}{\partial X_2} \frac{dX_2^*}{dX_1}$
iii) If $_2 \le X_2$, then $X_2^* = \underline{X}_2$, $\partial J / \partial X_1 = -\gamma$, $\frac{dX_2^*(X_1)}{dX_1} = 0$,

which can be illustrated in a diagram



Obviously, $\partial J / \partial X_1$ is not a concave function, unless $\gamma = -\infty$, but that corresponds to the case we just analyzed, i.e. the pure irreversibility case. As $\partial J / \partial X_1$ is neither convex nor concave as a function of X_1, it follows from Epstein's theorem that there exist information schemes Y, Y', and Y" and a random variable Z such that both Y' and Y" are more informative than Y and such that the optimal X_1' for Y' exceeds X_1 (the optimal choice for Y) and the optimal X_1" for Y" is less than X_1. Thus, it is impossible to say anything globally on the existence of the "irreversibility effect".

However, if $\partial V / \partial X_2$ is linear in X_2, $\partial J / \partial X_1$ is convex in the interval [0, X_2]. Let X_1' be the optimal choice of resource use in period 1 if the information scheme is Y' and let X_1" be the corresponding choice if the scheme is Y and assume Y' is more informative than Y. Furthermore, assume X_1' belongs to the interval [0, $\overline{X_2}$]. Then, if X_1 would have been restricted to that interval, the optimal choice would still have been X_1. It now follows from Epstein's theorem that

(3.8.5) $$X_1' \geq X_1".$$

Thus, if the initial resource use is small enough, then the prospect of getting more information in the future will increase the initial use.

In the same way, it is seen that $\partial J / \partial X_1$ is concave in the interval so that if X_1' is in this interval,

$$X_1' \leq X_1".$$

Finally, it follows that if X_1' belongs to $[\overline{X}_2, \underline{X}_2]$,

$$X_1' = X_1",$$

as $X_2^* = X_1$ on that interval.

Thus, if the initial resource use is larger than $\underline{X}_2$, then the prospect of more information will reduce the initial use, while if the initial resource use is smaller than $\overline{X}_2$ the prospect of more information will increase the initial resource use. We can therefore conclude by stating the following theorem.

Theorem

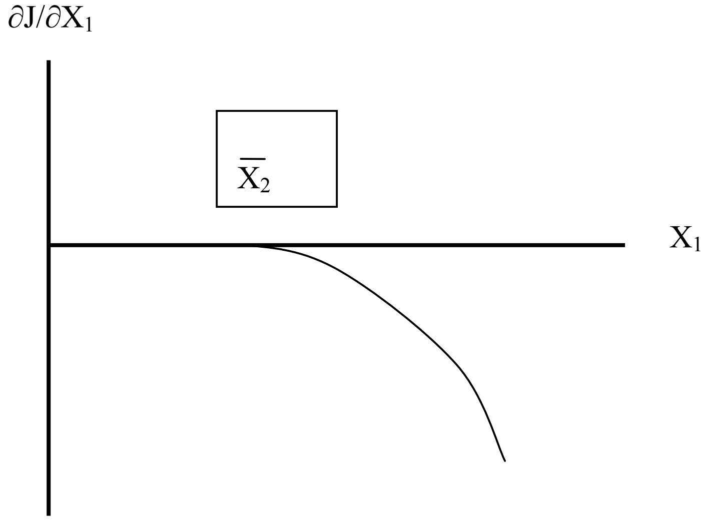
If it is possible to restore the resource according to the cost function defined in (), then for "small" initial resource use, an increase in expected forthcoming information will increase that initial resource use, while if the initial resource use is "large", more expected information will reduce the initial resource use.

This result may have an implication for the current discussion on global warming and emissions of green house gases. It has been claimed that the uncertainty about future impacts from climate change should imply that we would reduce the emissions more than would be desirable if no future information is forthcoming. However, the opposite has also been argued, and it may be that this theorem explains why serious scholars have come to such different conclusions.

Now, the theorem is a result of the discontinuity of the cost function. If we modify the cost function, a different result will emerge. Assume then that the cost function can be written

$$C(X_1, X_2) = \omega(X_1 - X_2) \tag{3.8.6}$$

where $\omega(.)$ is strictly concave, $\omega(0) = 0$, $\omega'(0) = 0$, and $\omega(t) = 0$ for t<0. The $\partial J \,/\, \partial X_1$ curve will now look like the curve in the following diagram:



It is now clear that the curve is concave and that the irreversibility effect is global.

Theorem *If the cost function looks like the diagram above and with the properties described above, there will be a global irreversibility effect.*

3.9 The value of information

We can now define the value of one information scheme relative to another, conditional on the decision resource use in the first period as

(3.9.1) $$V(X_1, Y", Y') = \sum_j q_j" J(X_1, \pi_j") - \sum_j q_j' J(X_1, \pi_j')$$

If Y" is more informative than Y', we know by definition that V(X $_1, Y', Y") > 0.$ V gives a measure of how much the expected utility from a resource use equal to X_1 will increase, if the forthcoming information changes from Y' to Y". It is natural to define a zero point for this value by choosing the case of no forthcoming information as a reference point. Thus the value of information Y, conditional on the resource use X_1 in the first period as

(3.9.2) $$V(X_1, Y) = \sum_j q_j J(X_1, \pi_j) - \sum_j q_j \max_{X_2} \sum_i r_i U(X_1, X_2, z_i).$$

But, as $\sum_j q_j = 1,$ we have

(3.9.3) $$V(X_1, Y) = \sum_j q_j J(X_1, \pi_j) - \max_{X_2} \sum_i r_i U(X_1, X_2, z_i).$$

In particular, the value of perfect information Y, conditional on the resource use X_1 in the first period is

(3.9.4) $$\sum_i r_i \max_{X_2} U(X_1, X_2, z_i) - \max_{X_2} \sum_i r_i U(X_1, X_2, z_i).$$

The unconditional value of information from the scheme Y is defined as

(3.9.5) $$W(Y) = \max_{X_1} \sum_j q_j J(X_1, \pi_j) - \max_{X_2} \sum_i r_i U(X_1, X_2, z_i).$$

It is easily seen that W(Y) is always non-negative and strictly positive whenever Y is strictly more informative than no information at all, and $\sum_j q_j J(X_1, \pi_j)$ has a unique maximum as a function of X_1. Assume now that there is a decision maker that does not take the possibility of future information into account. He will thus solve

(3.9.6) $$\max_{X_1, X_2} \sum_i r_i U(X_1, X_2, z_i)$$

However, if he would be paid a subsidy equal to the value of the forthcoming information

(conditional on his [1]choice of X_1 he would solve

(3.9.7) $$\max_{X_1, X_2} \left\{ \sum_i r_i U(X_1, X_2, z_i) + V(X_1, Y) \right\} = \max_{X_1} \sum_j q_j \max_{X_2} \sum_i \pi_i U(X_1, X_2, z_i).$$

Thus, a decision maker could be made to take forthcoming information into account by being subsidized with an amount equal to the value of information conditional on his choice of resource use in the first period.[13] However, if the decision maker is rational, he should of course have taken this information into account when he makes the decision and if that is the case, the subsidy would only distort the decision. Basically, there is no need for a subsidy, because the only kind of market failure that would cause an individual from considering the possibility of future information is irrationality. This kind of market failures cannot be solved by subsidies.

3.10 Uncertainty about irreversibility

Assume now that we don't know whether a decision today will have as a consequence an irreversible change in an environmental resource. Let us therefore assume that there is a positive probability p that the change will be irreversible and a corresponding positive probability 1-p that the change is reversible. In terms of the notations used in previous sections, the optimization problem is

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[13] The equivalence of option value and the value of information was first suggested by Conrad (1980). Fisher and Hanemann (1986) and Hanemann (1989) subsequently showed that option value is in fact a conditional value of information, as in the text, and that the conditional value is (not strictly) greater than the unconditional value.
]

$$(3.10.1) \max_{X_1}\left\{U(X_1+(1-p)\sum_j q_j \max_{X_2}\sum_i \pi_{i,j} v(X_2,z_i)+p\sum_j q_j \max_{X_{2}\geq X_1}\sum_i \pi_{i,j} v(X_2,z_i)\right\}$$

The solution to this problem will be compared first, with the solution to the problem when irreversibility is not expected and second with the solution to the problem when it is known for sure that the change is irreversible. The first of these problems is to determine the solution to

$$(3.10.2) \qquad \max_{X_1}\left\{U(X_1+\sum_j q_j \max_{X_2}\sum_i \pi_{i,j} v(X_2,z_i)\right\},$$

and the second problem is exactly the one we have discussed in previous sections. Denote the solution to the first problem X_1^r and the solution to the second problem X_1. We know that if more information is expected to come forth in the future,

$$X_1^r \geq X_1^i.$$

Let $\overline{X}_1$ be the solution to the problem when irreversibility is uncertain. Let us for simplicity assume an interior solution. For any choice of X_1 we have for each j the optimal X_{2j} defined by

$$(3.10.3) \qquad \max_{X_{2,j}\geq X_1}\sum_i \pi_{i,j} v(X_{2,j},z_i).$$

The sum is a linear combination of concave functions and is therefore concave. The optimal X_{2j} will obviously be a function of X_1 : $X_{2,j} = X_{2,j}^i(X_1)$. Let $X^r{}_{2,j}$ be the corresponding solution when there is no irreversibility constraint. If there is no such constraint, $\frac{dX_{2,j}^r}{dX_1} = 0$, otherwise $\frac{dX_{2,j}^r}{dX_1} = 1$. The assumption of an interior solution now yields (suppressing the random variable z_i)

$$\frac{\partial U(\overline{X}_1)}{\partial X_1}+\sum_j q_j \sum_i \pi_{i,j}\frac{\partial v}{\partial X_2}\frac{dX_2^i}{dX_1} = 0,$$

or

$$(3.10.4) \qquad \begin{aligned} \frac{\partial U(X_1^i)}{\partial X_1} &= -\sum_j q_j \sum_i \pi_{i,j}\frac{\partial v}{\partial X_2}\frac{dX_2^i}{dX_1} \geq \\ -p\sum_j q_j \sum_i \pi_{i,j}\frac{\partial v}{\partial X_2}\frac{dX_2^r}{dX_1} &= \frac{dU(\overline{X}_1)}{dX_1} \geq 0 = \frac{dU(x_1^r)}{dX_1} \end{aligned}.$$

The concavity of the U-function now yields that

$$(3.10.5) \qquad X_1^i \leq \overline{X}_1 \leq X_1^r.$$

Thus, we have reached the intuitively obvious but very important conclusion that when it is not known for sure that a change is reversible or not, it is better to be cautious and not

undertake as much development as would have been optimal if the change had known to be reversible. Note that this conclusion is not dependent on linearity, or a binary choice or even on the possibility of forthcoming information. Whenever, one is uncertain about the reversibility of a change, one should be cautious and not undertake as big change one would have had desired if the change had been known with certainty to be reversible.
If it not known whether a change is going to be irreversible or not, it is still beneficial to be cautious in that one should not undertake as big change as one would have had desired if the change had been known to be reversible. It is easily seen from the inequality above that the optimal amount of development will decrease as the probability of irreversibility goes up.

Finally, it is easily seen that if the utility function is linear or if there is a binary choice, the optimal amount of development will be non increasing with the probability of an irreversible change.

3.11 Option values

Let us now try to integrate the model with temporal resolution of uncertainty which we have been discussing in this part with the discussion in the first part on option prices and option values. Consider the situation where Q is the measure of environmental resource use and which can take two values Q', implying that the natural asset is preserved, and Q'' that it is developed and irrevocably lost. The benefits in the present period are known with certainty and are given by the indirect utility function

$$U(W^1, Q^1)\,, \tag{3.11.1}$$

where W^1 is wealth in the first period and Q^1 can take the two values Q' or Q''. The present value of future net benefits, given that state i occurs is

$$v^i(W_i, Q^2)/(1+\delta_{i)}\,, \tag{3.11.2}$$

where W_i is the future wealth if state i occurs, v^i is the utility function in state i, δ_i is the discount rate in state i, and Q^2 takes the values Q' or Q''. The total present value of benefits, given that state i occurs is

$$U(W^1, Q^1) + \frac{v^i(W_i, Q^2)}{1+\delta_i}\,, \tag{3.11.3}$$

where $Q^2 = Q^1$ if $Q^1 = Q''$ (the irreversibility assumption).Assuming the same information structure as in the previous section, the decision problem can be formulated

$$\max_{Q^1}\left\{U(W^1, Q^1) + \sum_j q_j \max_{Q^2} \sum_i \pi_{i,j} v^i(W_i, Q^2)/(1+\delta_i)\right\} \tag{3.11.4}$$
$$\textit{subject to } \mathrm{Q}^2 = Q'' \text{ if } \mathrm{Q}^1 = Q''.$$

The maximum price, CV, the individual would be willing to pay for keeping the option of deciding in the next period the use of the resource is given by

(3.11.5)
$$U(W^1 - CV, Q') + \sum_j q_j \max_{Q^2} \sum_i \pi_{i,j} v^i(W_i, Q^2)/(1+\delta_i) =$$
$$= U(W, Q") + \sum_j q_j \sum_i \pi_{i,j} v^i(W_i, Q")/(1+\delta_i).$$

The state contingent benefits CV_i of preserving the asset for at least one period are given by

(3.11.6)
$$U(W^1 - CV_i, Q') + \max_{Q^2} v^i(W_i, Q)/(1+\delta_i) =$$
$$= U(W^1, Q") + v^i(W_i, Q")/(1+\delta_i).$$

Thus,

(3.11.7) $$v^i(W_i, Q") = U(W^1 - CV_i, Q') + \max_{Q^2} v^i(W_i, Q)/(1+\delta_i) - U(W^1, Q"),$$

and substituting this into 3.11.5 gives

(3.11.8)
$$U(W^1 - CV, Q') + \sum_j q_j \max_{Q^2} \sum_i \pi_{i,j} v^i(W_i, Q^2)/(1+\delta_i) =$$
$$\sum_j q_j \sum_i \pi_{i,j} U(W^1 - CV_i, Q') + \sum_j q_j \sum_i \pi_{i,j} \max_{Q^2} v^i(W_i, Q)/(1+\delta_i).$$

This simplifies to (remembering that $\sum_j q_j \pi_{i,j} = r_i$)

(3.11.9)
$$U(W^1 - CV, Q') - \sum_i r_i U(W^1 - CV_i, Q') =$$
$$= \sum_j q_j \left\{ \sum_i \pi_{i,j} \max_{Q^2} v^i(W_i, Q)/(1+\delta_i) - \max_{Q^2} \sum_i \pi_{i,j} v^i(W_i, Q^2)/(1+\delta_i) \right\},$$

or

(3.11.10) $$U(W^1 - CV, Q') - \sum_i r_i U(W^1 - CV_i, Q') \leq 0.$$

If, as we assume, the utility function U is concave, it follows that

(3.11.11) $$\sum_i r_i U(W^1 - CV_i, Q') \leq U(W^1 - \sum_i r_i CV_i, Q'),$$

and finally

(3.11.12) $$U(W^1 - CV, Q') \geq U(W^1 - \sum_i r_i CV_i, Q'),$$

or

$$CV \leq \sum_i r_i CV_i.$$

Thus, the option value as defined by Weisbrod, Krutilla, Cichetti and Freeman[14] will be positive!

Theorem *The option value as defined in section 2.6 will be non-negative if information is expected to come forward*.

[14]See Krutilla et.al (1972), and Weisbrod (1964)

4. Many Periods, Continuous Time, and Stochastic Processes

In this section we extend the analysis to many periods and continuous time. In doing so we also touch on stochastic processes that underlie the uncertainty we are dealing with. Section 4.1 presents a simple two-period model that forms the foundation for subsequent analysis of the multi-period and continuous cases. The model is a little different from those in the preceding section, but the basic idea is the same: how are first-period or current decisions affected by the presence of uncertainty about future costs and benefits and the prospect of resolution of the uncertainty over time. Section 4.2 extends the model to many periods. Particular attention is given to the "optimal stopping" problem. In each period, the decisionmaker is confronted with the alternatives of continuing – say to produce widgets, or to preserve a tract of forest land – and enjoying a flow of benefits, respectively profits or recreation, on the one hand, or stopping and receiving a termination value, respectively the scrap value of the widget plant or the proceeds from a timber harvest, on the other. Section 4.3 extends the multi-period model to treat the case of continuous time, including solution of the optimal stopping problem in this setting. Section 4.4 presents an empirical application: when, if ever, to cut an old-growth redwood forest in Northern California where the recreation values associated with preserving the forest evolve continuously according to a plausible and widely used stochastic specification, a geometric Brownian motion.

4.1 Investment under Uncertainty

Let us consider the general problem, as set out in Dixit and Pindyck (1994), of investment under uncertainty. A firm faces a decision of whether or not to make an investment, with a sunk cost of I, in a factory that will produce one widget per period forever. The current price of widgets is P_0 and, in the second period and thereafter, it will be either $(1 + u) P_0$, with probability q, or $(1 – d) P_0$, with probability $(1 – q)$. The expected present value of the return to the investment is then

(4.1.1) $$V_0 = P_0 + \left[q\left(1+u\right) P_0 + \left(1-q\right)\left(1-d\right) P_0 \right] / r,$$

where r is the discount rate. If $V_0 > I$, the investment will be made; otherwise, it will not. Letting Ω_0 denote the net payoff, we have

(4.1.2) $$\Omega_0 = \max\left\{V_0 - I, 0\right\}.$$

This is the standard present-value criterion and, as we shall see, is in fact equivalent to the second decision rule in the last section's model of the decision on environmental preservation.

Implicit in equation 4.1.2 is that the investment is considered only for the first period. Now, suppose that the opportunity will be available in the second period if it is not taken in the first. The present value of the return to the second-period investment is

$$(4.1.3) \qquad V_1 = \begin{cases} (1+u)\,P_0 + \dfrac{(1+u)\,P_0}{r} & \text{if } price = (1+u)\,P_0 \\ (1-d)\,P_0 + \dfrac{(1-d)\,P_0}{r} & \text{if } price = (1-d)\,P_0. \end{cases}$$

The net payoff, the outcome of a future optimal decision, called the continuation value, is

$$(4.1.4) \qquad F_1 = \max\left\{V_1 - I, 0\right\}.$$

What is the implication for the first-period decision? Notice that, although the second-period decision is made under certainty [by the start of the second period, the firm knows whether price is (1 + u) P_0 or (1 – d) P_0 and optimizes accordingly], from the perspective of the first period, V_1 and F_1 are uncertain. Then, the expected continuation value, from the perspective of the first period, is

$$(4.1.5) \qquad \begin{aligned} E_0\left[F_1\right] = {} & q \max\left\{(1+u)\,P_0 + \frac{(1+u)\,P_0}{r} - I, 0\right\} \\ & + (1-q)\max\left\{(1-d)\,P_0 + \frac{(1-d)\,P_0}{r} - I, 0\right\}. \end{aligned}$$

The net payoff to the investment opportunity presented in the first period, optimally taken (in the first period or the second), is

$$(4.1.6) \qquad F_0 = \max\left\{V_0 - I, \frac{1}{1+r}\,E_0\left[F_1\right]\right\},$$

where V_0 – I is the expected present value of the investment made in the first period and (1)/(1 + r) $E_0[F_1]$ is the (discounted) expected continuation value—what the firm gets if it does not make the investment in the first period.

The difference, $F_0 - \Omega_0$, can be interpreted as option value: the value of the option to postpone the investment decision. As Dixit and Pindyck point out, the investment opportunity is analogous to a call option on a share of stock. It confers the right to exercise an option to invest at a given price (cost of the investment) to receive an asset (the widget factory) that will yield a stream of uncertain future returns. At first blush this appears somewhat different from the interpretation of option value in the model of section 3.2 above as a conditional value of information, but in fact is just another side of the same coin. The new information has value only if an option to postpone exists, and the option to postpone has value due to the information that will come available (though, depending on how the problem is formulated, the option to postpone an investment can be valuable even in the absence of new information).

4.2 Multiperiod Case

To extend the analysis to many periods (again drawing on the definitive presentation in Dixit and Pindyck [1994, chapter 4]), we simply repeat the procedure, starting at the next-to-the-last period and working back period by period. We introduce some new notation, which will also be useful when we go to continuous time. A firm's current status is described by the state variable, x, where x_t is known but x_{t+1}, x_{t+2}, etc., are random variables (but all of the relevant information about the probability distribution is in x_t since x is assumed to be a Markov process). The firm's choices are described by the control variable, u. State and control variables affect immediate profit flow, $\pi_t\left(x_t, u_t\right)$. Then, using previously defined concepts,

$$(4.2.1) \qquad F_t\left(x_t\right) = \max_{u_t}\left\{\pi_t\left(x_t, u_t\right) + \frac{1}{1+r} E_t\left[F_{t+1}\left(x_{t+1}\right)\right]\right\},$$

where $F_t(x_t)$ is the expected net present value of all of the firm's cash flows when all decisions are optimal from t on.

This is the Bellman equation. Bellman's Principle of Optimality is: An optimal policy is such that, whatever the initial action, the remaining choices constitute an optimal policy for the subproblem starting at the state that results from the initial action. In the two-period case, immediate investment ($u_0 = 1$) gave $V_0 - I$ and waiting ($u_0 = 0$) had only a continuation value. The choice was thus a binary one (invest or not at a fixed level). This is a special case of the Bellman equation.

Optimal Stopping

Another, more general, example of a binary choice is optimal stopping. In each period either stop and get a termination payoff, Ω, or continue with a current period profit plus an expected continuation value. Then, the Bellman equation becomes

$$(4.2.2) \qquad F_t\left(x_t\right) = \max\left\{\Omega\left(x_t\right), \pi\left(x_t\right) + \frac{1}{1+r} E_t\left[F_{t+1}\left(x_{t+1}\right)\right]\right\}.$$

The two-period investment problem is an optimal stopping problem, where the period 0 termination payoff is $V_0 - I$, and the period 0 profit from continuing (not making the investment) is 0. The control variable, u in the Bellman equation, is implicitly 0 or 1 in an optimal stopping problem, such as the two-period problem. For example, in that problem, choice of u_0 is either 0 (do not make the investment and get only the continuation value, $E_0[F_1]/1 + r$) or 1 (make the investment and get the termination payoff, $V_0 - I$).

In the more general optimal stopping problem just above, $u_t = 0 \Rightarrow$ do not stop, i.e., stay in business and get $\pi(x_t) + \frac{E_t\left[F_{t+1}(x_{t+1})\right]}{1+r}$, and $u_t = 1 \Rightarrow stop$ and get $\Omega(x_t)$. The point is that an optimal stopping problem involves a binary choice, just two alternatives, that we can explicitly specify (and that implicitly correspond to u = 0 and u = 1). This is often the way problems arise, as in the choice of environmental preservation or development.

Multiperiod Solution

To solve the general multiperiod problem, start at the last period, T. The firm gets a termination payoff, $\Omega_T(x_T)$. Then, at period T – 1,

(4.2.3) $$F_{T-1}(x_{T-1}) = \max_{u_{T-1}} \left\{ \pi(x_{T-1}, u_{T-1}) + \frac{1}{1+r} E_{T-1}\left(\Omega_T(x_T)\right) \right\}.$$

The decision then moves back to period T – 2, where

(4.2.4) $$F_{T-2}(x_{T-2}) = \max_{u_{T-2}} \left\{ \pi(x_{T-2}, u_{T-2}) + \frac{1}{1+r} E_{T-2}\left(F_{T-1}(x_{T-1})\right) \right\}.$$

The essential idea is to split the sequence of decisions into two parts: the immediate choice, and remaining decisions summarized in the continuation value. To find the optimal sequence, work backward. At the last decision point, make the best choice and get the continuation value (F_1 in the two-period case). Then, in the preceding period, we know the expected continuation value ($E_0[F_1]$) and can make the best choice (of the current control variable, here the decision of whether or not to invest).

4.3 Continuous Time and Stochastic Processes

Again drawing on Dixit and Pindyck (1994, chapters 3 and 4), we now consider the case of continuous time. Many economic and environmental processes do, in fact, evolve continuously rather than in discrete steps. Further, there are mathematical advantages to working in continuous time. Suppose that each period is of length Δt. Now, interpret $\pi(x, u, t)$ as the rate of profit, so profit over the interval Δt is $\pi(x, u, t)\ \Delta t$. Similarly, r equals the discount rate per unit time, so discounting over Δt is $\frac{1}{1+r\Delta t}$. The Bellman equation becomes

(4.3.1) $$F(x,t) = \max_u \left\{ \pi(x,u,t)\,\Delta t + \frac{1}{1+r\Delta t} E_t\left[F\left(x_{t+\Delta t}, t+\Delta t\right)\right] \right\}.$$

Multiplying both sides by $(1+r\Delta t)$, dividing by Δt, and letting $\Delta t \to 0$, we obtain

(4.3.2) $$rF(x,t) = \max_u \left\{ \pi(x,u,t) + \frac{1}{dt} E\left[dF\right] \right\}.$$

On the left-hand side, we have the opportunity cost of holding the asset for a unit of time, rF; on the right-hand side, the immediate payout or dividend from the asset plus the expected rate of capital gain.

The limit on the right-hand side depends on the expectation corresponding to the random $x_{t+\Delta t}$. One very important and widely used stochastic process (a variable that evolves over time in a way that is at least in part random) that allows such a limit in a form conducive to further analysis is the Ito process, a generalization of the basic Wiener process. If z(t) is a Wiener process, then an increment, dz, can be represented as $dz = \varepsilon_t \sqrt{dt}$, where ε_t is a normally distributed random variable with a mean of zero and a standard deviation of one. Important properties that follow from the definition are that $E(dz) = 0$ and variance (dz) = dt. Now, let the random variable, x(t), be an Ito process, defined such that an increment, dx, is represented as

(4.3.3) $$dx = a(x,t)\,dt + b(x,t)\,dz,$$

where a(x, t) and b(x, t) are known (nonrandom) functions and dz is the increment of a Wiener process. Ito's Lemma, a Taylor-series expansion of F(x, t), gives

(4.3.4)

$$dF = \left[F_t(x,t) + a(x,u,t)\, F_x(x,t) + \frac{1}{2} b^2(x,u,t)\, F_{xx}(x,t) \right] dt + b(x,u,t)\, F_x(x,t)\, dz$$ and

E[dF] = $\left[F_t(x,t) + a(x,u,t)\, F_x(x,t) + \tfrac{1}{2} b^2(x,u,t)\, F_{xx}(x,t) \right]$ dt since E[dz] = 0.

Substituting back in the Bellman equation, we obtain

(4.3.5)
$$rF(x,t) = \max_u \left\{ \pi(x,u,t) + F_t(x,t) + a(x,u,t)\, F_x(x,t) + \frac{1}{2} b^2(x,u,t)\, F_{xx}(x,t) \right\}.$$
We can express the optimal u as a function of x, t, F_t, F_x, and F_{xx} and the various parameters in π(., ., .), a(., ., .) and b(., ., .), substitute back into the Bellman equation, and get a partial differential equation of the second order with x and t as the independent variables. If the functions π, a, and b do not depend on time, then F does not either. Then, the Bellman equation becomes

(4.3.6) $$rF(x) = \max_u \left\{ \pi(x,u) + a(x,u)\, F'(x) + \frac{1}{2} b^2(x,u)\, F''(x) \right\}.$$

Again, substituting the optimal u in the Bellman equation, we now have an ordinary differential equation with x as the independent variable.

Optimal Stopping and Boundary Conditions

The solution will depend on boundary conditions in specific applications. There is one important class of problems for which we can state boundary conditions: optimal stopping, or binary decisions, with an Ito process.

The Bellman equation (in continuous time) is:

(4.3.7) $$F(x,t) = \max \left\{ \Omega(x,t),\ \pi(x,t)\, dt + \frac{1}{1 + r\, dt} E\left[F(x + dx, t + dt) \right] \right\}.$$

In the continuation region, the second term on the right-hand side is larger. Expanding it by Ito's Lemma, as before,

(4.3.8) $$rF(x,t) = \pi(x,t) + F_t(x,t) + a(x,t)\, F_x(x,t) + \frac{1}{2} b^2(x,t)\, F_{xx}(x,t).$$

We have "maximized out" u, choosing to continue, not terminate. This holds for x > x*(t), where x*(t) is the critical value of x with continuation optimal on one side (x > x*(t)) and stopping on the other.

There are two boundary conditions for x*(t). The first is the value matching condition. From the Bellman equation, we know that in the stopping region we have F(x, t) = Ω(x, t) so, by continuity, we can impose the condition

(4.3.9) $$F\left(x^*(t), t\right) = \Omega\left(x^*(t), t\right)$$

at the boundary of the stopping region. The second boundary condition is the smooth pasting condition. The values F(x, t) and Ω(x, t) should meet tangentially at the boundary, so

(4.3.10) $$F_x\left(x^*(t), t\right) = \Omega_x\left(x^*(t), t\right).$$

This condition is proved by contradiction in Dixit and Pindyck (1994, Chapter 4, Appendix C).

4.4 An Application: When to Cut an Old-Growth Redwood Forest

This is another problem of optimal forest use, here the Headwaters Forest, a large stand of privately owned redwood in Northern California which has been the subject of negotiation between the owner, who has plans to cut, and agencies of both state and federal governments, which would like to see the area preserved. A recent analysis by Conrad (1997) considers the question of when, if ever, it would be optimal (from a social point of view) to cut the Headwaters Forest. Conrad formulates the question as one of optimal stopping in continuous time. We can terminate at any time, harvest and get a payoff N (which corresponds to Ω, but is not in this application a function of a state variable or time) or we can continue and get a flow of amenity services (flood control, erosion control, wildlife habitat, recreation, etc.), A = A(t), where

(4.4.1) $$dA = \mu A\, dt + \sigma\, A\, dz,$$

i.e., A is a stochastic process of a particular type: a geometric Brownian motion. Note that this can be considered a special case of the Ito process defined earlier, where μ is the instantaneous drift rate and σ is the instantaneous variance rate. The geometric Brownian motion is very widely assumed in stochastic analysis of environmental and resource problems. It has the virtue of simplicity, and seems plausible, in this case at least, given the upward trend in amenity value, recreational use, etc., and increasing uncertainty about value as we look further into the future [note that $E\left(dA\right) = \mu A\, dt$ and variance (dA) = $\sigma^2\, A^2\, dt$].

This is an optimal stopping problem in continuous time. The Bellman equation is

(4.4.2) $$V\left(A\right) = \max\left\{N,\, A\, dt + \frac{1}{1+\delta\, dt}\, E\left[V\left(A + dA, t + dt\right)\right]\right\}.$$

Expanding by Ito's Lemma, in the continuation region, (4.4.3) $$\delta V(A) = A + \mu A V'(A) + \frac{1}{2}\sigma^2 A^2 V''(A).$$

This is a special case of the general optimal stopping condition, where the value function (V) is a function of just one variable (A). This is a second-order ordinary differential equation. Conrad shows that the solution is

(4.4.4) $$V(A) = kA^{-\alpha} + \frac{A}{\delta - \mu},$$

where α is a constant that depends on μ, σ, and δ and α > 0 if δ > μ. If $\delta \le \mu,$ it will never be optimal to cut. Therefore, we focus on the case where δ > μ. The first term on the right-hand side is the value of having an option to cut. As $A \to \infty, kA^{-\alpha} \to 0$, as it should. The second term on the right-hand side is the value of never cutting, the flow of benefits A divided by the adjusted discount rate, δ – μ. The value-matching condition is

$$V(A^*) = N,$$

and, substituting for V(A*),

(4.4.5) $$kA^{-\alpha} + \frac{A}{\delta - \mu} = N.$$

The smooth-pasting condition is

$$V'(A^*) = 0,$$

since N does not depend on A, yielding

(4.4.6) $$-\alpha\, kA^{-(\alpha+1)} + \frac{1}{\delta - \mu} = 0.$$

These two equations can be solved for the two unknowns, k and A*. Solving the smooth-pasting condition for k,

(4.4.7) $$k = \frac{A^{\alpha+1}}{\alpha(\delta - \mu)}.$$

Substituting this in the value-matching condition, we obtain

(4.4.8) $$A^* = \frac{\alpha(\delta - \mu)N}{\alpha + 1},$$

where A* is the critical amenity value, the level at which the decision maker is indifferent between preserving and cutting.

In order to obtain a numerical solution, we need to estimate μ and σ as specified in equation 4.4.1; assume something about the discount rate, δ; calculate α (from μ, σ, and δ as given in Conrad, p. 99); and substitute the value for N ($550 x 10^6). Conrad estimates μ and σ from data on visitation rates to Redwood National Park, near Headwaters, on the assumption that amenity value, A, is proportional to visitation, R. Visitation data from 1976-1995 are consistent with geometric Brownian motion, with μ = 0.05 and σ = 0.10. If δ = 0.05, A* = 0 (never cut because $\delta \le \mu$). If δ = 0.06, A* = $5.008 x 10^6—two orders of magnitude less than N (remember, A* is annual value and N is present value).

Repeated harvests do not add much to the problem, given the great value of old-growth redwood and the length of time required to grow a new, mature stand. Conrad and Ludwig (1994) have estimated elsewhere that the value of future harvests is often less than 2 percent of the value of the old-growth harvest. If N = $600 x 10^6, A* = $5.463 x 10^6, which is very close to the $5.008 x 10^6 for N = $550 x 10^6. This is for a nearly 10 percent increase in N, which is much greater than 2 percent.

The impact of a change in the variance can, however, be substantial. The instantaneous variance rate is σ = 0.1 in the base case. If instead $\sigma = 0.2$, A* = $3.988 x 10^6; preserving the old-growth forest is more attractive (requires a lower amenity value) as the amenity value becomes more volatile. The interpretation is that, if the forest is preserved, it is possible to benefit from an above-average increase in amenity value, while retaining the option to cut in the event of a below-average increase, or decline, in amenity value.

Appendix A
On the equivalence of two representations of environmental changes.

Assume that λ ' has a density ν (i) (Radon-Nikodym derivative) [Halmos65] with respect to λ so that

$$\lambda_i' = \nu(i)\lambda_i$$

Assume furthermore that Q $_i$ is increasing in i, U $_i$ (W $_i$, Q) is increasing in Q and v(i) > 0.
Then define

$$Q_i' = U^{i^{-1}}W_i, U^i(W_i, Q_i v(i))$$

where U $^{i^{-1}}$ is the inverse function to $\partial U^i / \partial Q$
Then it is easily seen that

$$\int U^i(W_i, Q_i')d\lambda_i = \int U^i(W_i, Q_i)d\lambda_i'$$

Moreover, assume that the policy change $\lambda \rightarrow \lambda'$ means an increase in the probability of states with a high Q, i. e. that $\lambda_i \geq \lambda_i'$. This in turn implies that v(i) $\geq$ 1 and it follows that

$$Q_i' = U^{i^{-1}}(W_i, U^i(W_i, Q_i v(i))) \geq U^{i^{-1}}(W_i, U^i(W_i, Q_i)) = Q_i$$

so that the new Q' variable corresponds to a higher (or unchanged) Q in each state, a result we would demand of an economic reasonable representation.
The converse proposition follows from a simple variable substitution, i.e. let Q' = T(Q) where T is measurable. Then it follows from Halmos[15] that

$$\int U^i(W_i, T(Q_i))d\lambda_i = \int U^i(W_i, Q_i)d(\lambda_i T^{-1})$$

and we choose d $\lambda_i' = d(\lambda_i T^{-1})$.

Appendix B, A different notion of more

[15]Halmos 1965, &32

informative

Epstein´s analysis was based on the notion of "value of information". Freixas and Laffont (1984) developed a parallel and equivalent analysis based on a different notion of more informative. Let z be as before the random variable characterizing the preferences. Let z be defined on a probability space (Ω, F, μ) (Ω is the set of all outcome, F is the field of subsets on which the probability measure μ is defined. Any partition $\mathbb{Z}$ of Ω (compatible with F, i.e. if $S \in \mathbb{Z}$ then $S \in$ F) corresponds to an information structure, that is the decision maker knows whether the true state is a point in a set in $\mathbb{Z}$ or not. Briefly, S is an information set. It is now obvious that more informative can be defined by comparing partitions $\mathbb{Z}, \mathbb{Z}'$, etc The finer the partition is ($\mathbb{Z}'$ is finer than $\mathbb{Z}$ if $\mathbb{Z} \subset \mathbb{Z}'$), the more information the decision maker has. Green and Stokey (1978) proved that this definition of "more informative" is equivalent to Blackwell's definition (1951).
Let us assume that the objective function of the decision maker is linearly separable between the two time periods:

$$U(X_1) + V(X_2, z)$$

The decision on X_1 will give rise to the feasibility set $X_2(X_1) = \{X_2 ; X_2 \varepsilon X_1\}$ in the second period. Given X_1, the choice in the second period is determined from

$$max_{X_2 \in X_2(X_1)} E_\mu[V(X_2, z) | z \in S] = v_2(X_2, S)$$

Let $V_2(X_1, \mathbb{Z})$ be defined as

$$V_2(X_1, \mathbb{Z}) = E_\mu v_2(X_2(X_1), S).$$

Finally, let X_1* be the optimal first period choice when the partition is $\mathbb{Z}$, and $X_1'*$ when the partition is $\mathbb{Z}'$.
We can now state the main theorem derived by Freixas and Laffont:

Theorem: *Let $\mathbb{Z}'$ be a refinement of $\mathbb{Z}$. If V_2 is quasi-concave in X_1, then*

$$X_1*' \leq X_1*$$

This theorem is very similar to the one Epstein derived, difference being that here we assume that V_2 is quasi-concave while Epstein assumed U_2 is concave. It is easy to see that if U_2 is concave V_2 must be quasi-concave so that the Freixas and Laffont's theorem is slightly more general.

References

Albers, H.J., A.C. Fisher, & W.M. Hanemann, (1996), Valuation and management of tropical forests: Implications of uncertainty and irreversibility, *Environmental and Resource Economics* 8:39-61.

Albers, H. J., and M. J. Goldbach, (2000), Irreversible ecosystem change, species competition, and shifting cultivation, *Resource and Energy Economics* 22:261-280.

Arrow, K. J. & A.C. Fisher (1974) Environmental preservation, uncertainty, and irreversibility, *American Economic Review* 88: 312-320.

Arrow, K. J. & R. Lind (1972) Uncertainty and the evaluation of public investment decisions, *American Economic Review,* 60:

Bishop, R. (1982), Option Value: An exposition and extension, *Land Economics***,** 58**.**

Blackwell, D. (1951) Comparisons of experiments, in J. Neyman (ed.) *Proceedings of the Second Berkley Symposium on Mathematical Statistics and Probability.*

Bohm, P. (1975), Option demand and consumer's surplus: A comment, *American Economic Review* 65

Bradford, K. & Keleian, H. (1977), The value of information for crop forecasting in a market system *The Review of Economic Studies* 44

Cichetti, C. & A.M. Freeman III, (1971), Option demand and consumer surplus: Further comment, *Quarterly Journal of Economics* 85

Conrad, J.M, (1980), Quasi-option value and the expected value of information, *Quarterly Journal of Economics* 95:813-820

Conrad, J.M. (1997) On the option value of old-growth forest, *Ecological Economics* 22:97-102

Conrad, J.M. & D. Ludwig, (1994) Forest land policy: The optimal stock of old-growth forest, *Natural Resource Modeling* 8:27-45

Dasgupta, P. and K.-G. Mäler, eds. (2003) The Economics of Non-Convex Ecosystems, Kluwer Academic Publishers, Dordrecht

de Angelis, H., and P. Skvarca, (2003), Glacier surge after ice shelf collapse, *Science*.

Dixit, A.K. & R. S. Pindyck, (1994) *Investment under uncertainty,* Princeton University Press, Princeton, NJ

Epstein, L. (1980), Decision making and the temporal resolution of uncertainty, *International Economic Review* 63

Fisher, A.C. (1973), Environmental externalities and the Arrow-Lind public investment theorem, *American Economic Review* 63

Fisher, A.C. & W. M. Hanemann (1986), Option value and the extinction of species, in V.K. Smith, ed. *Advances in Applied Microeconomics* , pp. 169-190, JAI Press, Greenwich, CT

Freixas, X. &J.-J. Laffont (1984) On the irreversibility effect, in M. Boyer & R.E. Kihlstrom (eds.) *Baysean Models in Economic Theory*, North-Holland, pp. 105-114

Green J. and N. Stokey, (1978), "Two representations of Information Structures and Their Comparisons", Technical Report no. 271, IMSS, Stanford University

Halmos, P. (1965), *Measure Theory,* van Nostrand, New York

Hanemann, W.M. (1989), Information and the concept of option value, *Journal of Environmental Economics and Management*, 16:23-37

Henry, C. (1974a) Investment decisions under uncertainty: The irreversibility effect, *Review of Economic Studies: Symposium on the Economics of Exhaustible Resources.*

Henry, C. (1974b) Option values in the economics of irreplaceable assets, *Review of Economic Studies: Symposium on the Economics of Exhaustible Resources*

Krutilla, J.V., Cicchetti, C.J., Freeman III, A.M. & Russel, C.S. (1972) Observations on the economics of irreplaceable assets, in A. Kneese & B. Bower (eds.) *Environmental Quality Analysis: Theory and Method in the Social Sciences*, John Hopkins University Press

Malinvaud, E. (1972) The allocation of individual risks in large markets, *Journal of Economic Theory*, 4

Marshak, J. & K. Miyasawa (1968), Economic comparability of information systems, *International Economic Review* 9

Pindyck, R.S. (2000*) Irreversibilities and the timing of environmental policy, Technical report,* MIT

Raiffa, H. (1968) *Decision Analysis: Introductory Lectures on Choices under Uncertainty,* Random House, New York

Rod, B.W. (1964) Collective-consumption services of individual consumption goods, *Quarterly Journal of Economics* 78

Scheinkman, J.A. & T. Zariphopoulou, *Optimal environmental management in the presence of irreversibilities,* Technical Report: Nota di Lavoro 15.96, FEEM, Milan

Schmalensee, R. (1972) Option demand and consumer's surplus: valuing price changes under uncertainty, *American Economic Review* 62

Schultz, P. A., and J. F. Kasting, (1997), Optimal reductions on co_2 emissions, *Energy Policy* 25: 491-500.

Tongpan, S., T. Panayotou, S. Jetanavanich, K. 'Faichampa, & C. Mehl, (1990), *Deforestation and poverty: Can commercial and social forestry break the vicious circle?* Research Report No.2, 1990 TDRI Year-End Conference

Ulph, A.M. and D. Ulph, (1997) Global warming, Irreversibility and Learning*, Economic Journal* Vol. 107, pp.636-650.

Weisbrod, B. (1964) Collective-consumption services of individual consumption goods, *Quarterly Journal of Economics* 78:471-477.

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[i]A proof of the convexity of the maximum function is given in Albers, Fisher, and Hanemann (1996).