Assessing Sustainable Development in a DICE World

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Abstract

This paper investigates a method for assessing sustainable development under climate change in the Dynamic Integrated model of Climate and the Economy (DICE-2007 model). The analysis shows that the results, with respect to sustainability are highly sensitive to the calibration of the social welfare function. When revising the social welfare parameters of the DICE-2007 model to the alternative parametrization approach, used in the DICE-(1994,1999) model, it is only the former that upholds a sustainable productive base. This finding implies that when recalibrating the social welfare function, to match historical rates of return on capital, this can result in inconsistent projections of future social welfare. The robustness of these results are investigated by imposing uncertainty, regarding key parameter estimates. This shows that the social welfare parameters along with total factor productivity growth are much more important as determinants of productive base sustainability than climatic parameters such as the damage or temperature sensitivity coefficients.

Keywords: Sustainable development, Climate Change, DICE-model, Productive base, Discounting.

JEL Classifications: Q01, Q54

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1 Introduction

Integrated assessment models have become well used tools among researchers when trying to estimate the costs associated with climate change (Cline, 1992; Nordhaus, 1994; Manne *et al.*, 1995; Hope, 2006). These general models describe the climate-economy interlinkages in terms of dynamic, global macroeconomic growth models that are coupled with a climate model, describing the effects of increasing greenhouse gas concentrations on temperature. The models typically assume rationality among economic agents and therefore take the normative approach of deriving optimal policies for the most efficient way of slowing climate change. This has resulted in varying recommendations and much debate regarding the urgency for climate change mitigation e.g. ranging from postponing mitigation for several more decades (Nordhaus, 2007), to spending roughly 2% of GDP on mitigation efforts as proposed in the *Stern Review* (Stern, 2007). The debate surrounding these models have had large policy implications, one example being the world's first long term legally binding framework to tackle the dangers of climate change, known as the *Climate Change Act 2008*[1]. As, these models are much referred to in the climate change debate it is troublesome that there is such a divergence in modelling outcomes. The purpose of this paper is therefore to shed some light on the matter by looking at what optimal consumption paths imply for the social-wellbeing of future generations and how these outcomes are affected by different policy regimes and the social-ethical assumptions undertaken. This is done by applying the concept of sustainable development to a neoclassical growth contexture.

Sustainable development is one of many ways in which we can compare social-wellbeing between different generations. The term became popular after a publication by the World Commission on Environment and Development, which commonly became known as the Bruntland Commission Report. In this report sustainable development was defined as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs".[2] Since then there have been many attempts to make this term operational in conventional theoretical models of the economy. Pezzey and Toman (2005) provide a good overview of some of these different economic interpretations of sustainable development. In this paper I will define sustainable development as non-declining social welfare. This definition relates to the idea that each generation should leave behind at least as large a *productive base* as they were given by their ancestors, meaning that each generation will have the same possibility to generate welfare as the generation be-

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[1] The *Climate Change Act 2008* is a legislation forcing a 80% reduction in greenhouse gas emissions in the UK by the year 2050. In the background documents several referrals are made to the *Stern Review*. `http://www.opsi.gov.uk/acts/acts2008/pdf/ukpga_20080027_en.pdf`

[2] World Commission on Environment and Devolopement (1987)

fore them.[3] The definition is mainly due to Dasgupta and Mäler (2000) and Arrow *et al.* (2003) and is formulated in terms of the Ramsey-Koopmans social welfare functional, where an arbitrary consumption path is denoted as sustainable if the time derivative of the social welfare functional is greater than or equal to zero.[4] As emphasized in Arrow *et al.* (2003) the concept of optimality differs from sustainability. Even under an optimal policy plan the realized consumption path could be rendered unsustainable if for example the pure rate of time preference, also commonly referred to as the utility discount rate, was set too high.[5] Therefore it is of interest to evaluate if the policy recommendations given by the integrated assessment models also pertain consumption paths that are consistent with sustainable development i.e. that they do not exhaust the economy's *productive base*. If this were the case, the policy recommendations given by the model would be steering us in a direction of impoverishment, without informing us that this was actually happening.

This paper makes a first pass at evaluating one of these integrated assessment model in terms of sustainability, namely the DICE-2007 model (Nordhaus, 2007).[6] The results show that although the DICE-2007 model proves to be *productive base* sustainable, this result remains highly sensitive to the specific discounting assumptions employed within the model. When the discounting assumptions used in the DICE-2007 model are compared to the alternative discounting approach, of the DICE-(1994,1999) model, it is only the former that manages to maintain a sustainable *productive base*. This finding implies that even though the parameters of the social welfare function are chosen consistently (i.e. to match historical rates of return on capital), with small implications for the saving rate, the social cost of carbon or the optimal carbon tax, the relative choices of these parameters still affect other aspects of the model. The robustness of these results are evaluated by introducing uncertainty regarding the most model sensitive parameter values, which to a large extent shape the dynamic structure of the model. The conclusions from the uncertainty analysis is that the most important parameter estimates determining whether the model will turn out to be *productive base* sustainable or not, are the social welfare parameters (i.e. the utility discount rate and the elasticity of marginal utility) along with the total factor productivity (TFP) growth rate. For example,

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[3] The term *productive base* refers not only to all sorts of physical, human and natural capital, but also to institutional arrangements and knowledge. Arrow *et al.* (2003) show that the maintenance of a productive base implies and is implied by non-declining social welfare.

[4] The equivalent notion for the discrete time case, is that social welfare should be non-declining between two periods of time.

[5] Arrow *et al.* (2003) also claim that it is possible to find that along an optimum path social welfare declines for a period and then increases thereafter, in which case the optimum programme does not correspond to a sustainable path locally, but does so in the long run.

[6] DICE is an acronym for the "Dynamic Integrated model of Climate and the Economy".

if the growth rate of TFP is set to zero, while leaving everything else unchanged, this would induce an unsustainable path in the DICE-2007 model. As argued by Dasgupta *et al.* (1999) and recently shown in Vouvaki and Xepapadeas (2008b) this assumption is far from implausible.

To my knowledge, this is the only study that has evaluated sustainability in terms of changing production possibilities within an integrated assessment model. Previous empirical analysis of *productive base* sustainability within theoretical frameworks include Vouvaki and Xepapadeas (2008a, 2009). Vouvaki and Xepapadeas (2008a) analyze the behavior of the sustainability criterion by empirically parameterizing a standard Solow model subject to a flow pollutant (SO_2) with estimates from the Greek economy. In Vouvaki and Xepapadeas (2009) they instead evaluate sustainability in a global macroeconomic growth model subject to the influence of a stock pollutant (CO_2) for 44 different countries. Their main empirical finding from the later paper is that a business as usual increase in CO_2 emissions will produce a negative measure of sustainability for most countries, while a constant level will yield a positive measure. Although, there are many structural differences between the model used in their analysis compared to the DICE model, the results from this paper still indicates that the major differences in outcomes could be due to different assumptions regarding key parameter estimates. For example, when implementing the choices of social welfare parameters used by Vouvaki and Xepapadeas (2009) in the DICE-2007 model, this produces an unsustainable production path.

The contribution of this paper is thus threefold. First, it shows how issues regarding sustainable development can be evaluated within an integrated assessment model framework. Second, it highlights which parameter estimates have the greatest effect on the evolution of the productive base. Third, it shows that model projections are sensitive to the relative choices of social welfare parameters, regardless whether these are calibrated to match historical rates of return on capital.

The rest of this paper is structured as follows: Section 2 describes the basic structure of the DICE-2007 model and how sustainable development can be evaluate within this framework. Section 3 presents result regarding sustainability for the different policy scenarios analyzed by Nordhaus (2007) and how these results are altered when the discounting assumptions of the DICE-(1994,1999) model are implemented. Section 4 evaluates how uncertainty regarding key parameter estimates effect the results with respect to sustainability. Section 7 concludes.

2 Sustainability Criterion in the DICE-model

The DICE model is one of the most accepted and widely known integrated assessment models for analyzing links and feedbacks between economic and climatic system. The DICE model was originally developed by Nordhaus (1994) and has since then been updated twice (Nordhaus and Boyer, 2000; Nordhaus, 2007); it is highly transparent and well documented with a freely available program code developed in the GAMS software. The purpose of the model is to combine knowledge from economic and climate sciences in order to derive insights into the costs and benefits of alternative policies for slowing climate change.

2.1 DICE model structure

The model assumes that economic and climate policies should be constructed in a way that maximizes the discounted population-weighted utility of per capita consumption over a 600 year time period. This optimization problem is solved by choosing the level of investment and emission control rate that maximizes the sum of discounted future utility (see equation A.1 of appendix), subject to economic and climatic conditions (A.2-A.21).[7] The model involves production of a single commodity, which can be used for either consumption or investment (A.7).

The aggregate output (A.3) of the model is produced using a Cobb-Douglas production function with an exogenous population growth (A.15) and a Hicks-Neutral technological change (A.16). The environmental damages and abatement costs (A.5-A.6) are assumed to be proportional to world output and the accumulation of capital (A.4), depends on investment decisions and the natural depreciation rate. The model incorporates a simple carbon cycle system, where carbon flows between three adjacent reservoirs consisting of the atmosphere, upper and lower oceans. The accumulation of greenhouse gases in the atmosphere leads to a warming of the surface through an increase in radiative forcing, which in turn leads to a rise in average global temperature levels (A.9-A.14).[8] The increase in global temperature levels results in physical damages that hinder future production possibilities e.g., through damages on agriculture, migration due to sea-level rise, adverse impacts on health, non-market damages and potential catastrophic impacts. It is also assumed that damages from small and gradual temperature increases are low, but that damages rise in a non-linear fashion (A.5).

Output is produced using energy from either a carbon based or a non-carbon based energy source. Energy use produces an externality in form of CO_2 emissions that together with the emissions from land use (A.19) change accumulates into the atmosphere (A.8-A.9) The amount of emissions originating from output depends on the level of carbon-saving technological change, which is modelled as exogenous (A.17). Production decisions are also based on the size of the abatement costs, which depends on the carbon-saving technology, the price of carbon-fuel replace-

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[7] The level of investment I_τ and emission control rate μ_τ constitute the control variables of this optimization problem.

[8] Other greenhouse gases are also included in the model as exogenous trends in radiative forcing (A.21).

ments (backstop technology) (A.18) and the global participation rate in mitigation efforts (A.6).

The amount of available fossil fuels is limited, which implies that there is an upper bound on emission levels (A.20). This assumption generates hotelling or scarcity rents for carbon based energy sources that can be of useful to describe the market path for emission reduction. This also implies that even in a laissez-faire economy there will be some amount of mitigation efforts if carbon fuels are scarce.

By modifying this general framework Nordhaus (2007) attempts to replicate and compare alternative environmental policies for tackling climate change. In total, 16 alternative policies are analyzed and compared with respect to the resulting economic and climatic outcomes that they produce.[9]

2.2 Sustainability

In order to evaluate whether the economic programmes analyzed within the DICE model framework are consistent with sustainable development, I will use a similar criteria for assessing sustainable development as was adopted by Dasgupta and Mäler (2000).[10] As opposed to optimality, which focuses on achieving a maximum present value flow of consumption over time, the sustainability criterion instead aims at evaluating whether the production possibility set is growing or declining. The main advantage of this method is that it can be incorporated into a general framework which is independent of whether the economies exhibit optimizing or non-optimizing behavior.[11] In mathematical terms the theory basically says that, a consumption path corresponds to a sustainable path at time t, if social welfare at time $t+1$ is not smaller than at time t, i.e. if $V(t+1) \geq V(t)$, where social welfare $V(t)$ can be defined as in expression (A.1):

$$V(t) = \sum_{\tau=t}^{T} (1+\rho)^{-(\tau-t)} U[c(\tau)] L(\tau) \qquad (1)$$

The value function $V(t)$ reflects social welfare as a function of a consumption and population growth at each moment in time. Our consumption possibilities will further depend on our production possibilities, which in turn is determined by our original endowment of manufactured capital, human capital, natural capital and knowledge, and also by the institutions governing the economy. All these factors put together can be said to constitute society's *productive base* (Dasgupta, 2001). If

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[9] A brief description of these policies is given in section 3, for further details see Nordhaus (2007).

[10] Dasgupta and Mäler (2000) developed their analysis in continuous time. Mäler (2001) provides a translation of basic theory into a discrete time setting.

[11] Dasgupta and Mäler (2000) defined a non-optimizing economy in the following way: a non-optimizing economy is an economy where the government whether by design or incompetence does not choose policies that maximize intergenerational welfare.

the institutional structure of the economy remains unchanged over time, the change in social welfare from one time period to the next, will be entirely determined by the changes in the *productive base*. Under these circumstances sustainable development at time t can be described as a pathway in which

$$V(\Phi(t+1)) \geq V(\Phi(t)), \quad \textit{at a given time } t \tag{2}$$

where $\Phi(t)$ denotes the state of the *productive base* at time t.[12] As shown in Arrow *et al.* (2003), this implies that at each period in time t the change in social welfare will equal genuine investment, which is defined as the accounting value of the rate of change in the productive assets of the economy. All assets which make up the economy's *productive base* will thus carry accounting prices, which are the weights determining, the direction and magnitude, resulting from increases in specific productive assets, such as for example manufactured capital, human capital, natural capital or technology. The sustainability criterion will thus also be a measure of how the economy's wealth is evolving.

2.3 Sustainability in the DICE model

Returning to the DICE model, a given set of parameter values, along with the emission control and saving rate associated with an optimal consumption path at time t and $t+1$, will imply that the value of the social welfare function $V(t)$ will be entirely determined by the state of its *productive base*. The *productive base* of the DICE model consists of initial values for all variables that determine the state of the system at any given moment of time. The vector of elements included in the *productive base* of the DICE-model, are presented in Table 6 of the Appendix. These are the variables that, as opposed to the parameters and constants of the model, evolve over time and therefore implicitly determine the level of feasible consumption at each point in time. Since the DICE model considers a time horizon of 600 years which is broken down into 60 time periods starting in 2005. Letting $\Phi(\cdot)$ denote the vector of elements given in Table 6, the sustainability criterion can thus be formulated as:

$$V(\Phi(2015)) \geq V(\Phi(2005)) \tag{3}$$

This means that sustainable development in the year 2005 is determined by the differences in the value of the sums of the maximized discounted future utility streams evaluated in the years 2015 and 2005, respectively. This evaluation is done for two sets of estimates for the pure rate of time preference and elasticity of marginal utility, used in the DICE-2007 model (section 3) and the DICE-(1994,1999) model (section 4), respectively.

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[12] It is also possible to formulate the much more demanding requirement for sustainable development, that this condition should be satisfied for all t.

3 Sustainability of policy scenarios and the role of the social welfare parameters

One of the major novelties with the DICE-2007 model is that the social welfare function has been revised with updated values for the utility discount rate (ρ) and the elasticity of marginal utility (η). In the new model, the level of utility discounting has been revised from an initial value of 3%, used in the DICE-(1994,1999) model to a lower value of 1.5%, while simultaneously raising the value for the elasticity of marginal utility from 1 (logarithmic) to a new value of 2. As explained in Nordhaus (2007): "this revision moves the model closer to one that displays intergenerational neutrality while maintaining the calibration of the model's rate of return on capital with empirical estimates". This argument stems from the famous Ramsey Rule which states that along an efficient and optimal economic programme the social rate of return on investment r will be given by:

$$r = \rho + \eta \frac{dc/dt}{c} \tag{4}$$

With perfectly functioning capital markets, no taxes and lack of divergence between private and social benefits, the social rate of return on investment will equal the private rate, implying that the market interest rate (return on capital) can serve as an appropriate proxy for social discounting.[13] Hence, these two parameters work in dissimilar ways; raising the value of η produces a more egalitarian outcome with increased intergenerational consumption smoothing while a lower value of ρ raises the value of future consumption streams. By recalibrating the model in this way Nordhaus finds a way to lower the value for the pure rate of time preferences (thus pleasing his critics that have pointed out the ethical dilemmas for high rates of utility discounting), while still maintaining the general model results. Table **??** lays out the sustainability results for the different policy scenarios when using a utility discount rate of 3% and a elasticity of marginal utility of 1.

3.1 General results

The sustainability criterion associated with each different policy scenario analyzed in Nordhaus (2007) are presented in table 1a. These policy scenarios are identical to the ones analyzed by Nordhaus. Table 1b shows the results of a separate run using the discounting assumptions following the *Stern Review*.[14] The numbers in parenthesis are the exact sustainability measures defined in social welfare terms where a relatively high (low) value is an indication of a higher (lower) level of sustainability.

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[13] See for example Arrow and Kurz (1970), Dasgupta and Heal (1979)

[14] The differences between this run and the last policy proposal in table 1a are explained in section 3.6.

The numbers before these are index numbers indicating policy performance relative to the *business as usual* (*250-year delay*) policy run. The first column contains the social welfare parameters used in the 2007 model with a utility discount rate (ρ) of 1.5% and a elasticity of marginal utility (η) of 2. The second column contains the social welfare parameters used in the 1994 and 1999 versions of the model.

Table 1a: Sustainability criteria for alternative policy scenarios

Discounting assumptions	DICE-2007 $\rho = 1.5\%,\ \eta = 2$	DICE-(1994,1999) $\rho = 3\%,\ \eta = 1$
Reference Scenarios		
250-year delay (BAU)	100 (21245)	100 (-107.14)
50-year delay	100.22 (21292)	100.43 (-106.68
Optimal	100.32 (21313)	100.66 (-106.43
Kyoto Protocol		
With United States	100.11 (21269)	100.23 (-106.89)
Without United States	100.02 (21249)	100.05 (-107.09)
Strengthened	100.24 (21295)	100.21 (-106.92)
Climatic constraints		
Concentration limits		
Limit to $1.5xCO_2$	92.06 (19559)	84.59 (-123.65)
Limit to $2xCO_2$	100.23 (21294)	100.49 (-106.62)
Limit to $2.5xCO_2$	100.32 (21313)	100.66 (-106.43)
Temperature limits		
Limit to 1.5°C	97.23 (20656)	93.10 (-114.53)
Limit to 2°C	99.52 (21144)	98.77 (-108.46)
Limit to 2.5°C	100.14 (21275)	100.26 (-106.86)
Limit to 3°C	100.25 (21300)	100.59 (-106.51)
Ambitious proposals		
Gore Proposal	98.82 (20995)	94.81 (-112.7)
Optimal (low-cost backstop)	101.92 (21653)	104.76 (-102.04)
Stern Proposal (dual-discount)*	101.98 (21453)	(98.92) -108.22

*See section 3.6 for details.

Table 1b: Sustainability criteria for the Stern Review

	$\rho = 0.1\%,\ \eta = 1$
Stern Review	1731

An immediate observation from table 1a is that all alternative policy scenarios analyzed in the 2007 model (column 1) have large positive sustainability measures while the social welfare parameters associated with the 1994 and 1999 model (column 2) generate negative sustainability measures. This is an illuminating result which indicates that the choices of these two key parameter estimates are very important in determining the model outcome with respect to sustainability and that a calibration approach which only focuses on maintaining a certain level of return on capital does not insure against possibly large impacts on the economy´s *productive base*. Further, as will be shown in the sensitivity analysis, the sustainability measure is more sensitive to the choice elasticity of marginal utility compared to the utility discount rate.

3.2 Reference scenarios

The first policy run analyzed is one where governments take no policy measures to internalize the costs of damages associated with greenhouse warming. In this policy run the market path for allocating carbon fuels is followed for 250 years, after which the world wakes up and optimizes its emission trajectory in light of climate damages.[15] This run corresponds to the *business as usual* (BAU) scenario which is also referred to as the *baseline* or *no controls* case. The baseline scenario can be compared to the *optimal* policy scenario where an optimal path for emission reduction is followed in order to maximize the value of net economic consumption.[16] Comparing the sustainability measures of these two we find that the *optimal* policy scenario produces a more sustainable consumption path in both columns. This is inline with what we would expect since the *optimal* policy run corresponds to the case where the best possible policy path is followed with regards to the economic, technological and geophysical constraints for the entire time horizon. Logically this should improve on the case where the market path is followed for 250 years with no consideration for damages inflicted by climate change. The reason for this is that when the market path is followed, one of the control variables (the emission control rate) is fixed within the optimization problem, implying that the optimal values cannot be chosen freely. This of course leads to an inferior solution implying that the *optimal* run is more sustainable. The same is true to that of the *50-year delay* run which is equivalent to the *250-year delay* except that the world wakes up earlier, which implies that it lies closer to the projections of the *optimal* policy run.

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[15] The market path implies that the optimal consumption path is chosen without consideration for the externalities associated with production. A more detailed description of the policy runs can be found in Nordhaus (2007), the time period 250 years is arbitrary and chosen for computational reasons, for example increasing the time span to 350 years has a very marginal effect.

[16] This policy scenario can be considered as having the normative perspective and would comply to the objectives of a benevolent social planner.

3.3 Kyoto Protocol

Concerning the three versions of the Kyoto Protocol runs they all produce sustainable results which supersede that of BAU. This is because the emission control rate chosen in this run corresponds better to that of the *optimal* policy run. We can also see that the version with the United States included generates a higher sustainability measure than the one without. This is a logical result indicating that a higher world participation rate in climate policies is more effective and hence increases sustainability. In the strengthened version of the protocol more countries are added gradually; they begin with a 10 percent emission reduction and reduce further with 10 percent every 25 years, which generates a more sustainable path than the other versions of the protocol. However, as can be seen in column 2 the equivalent result does not hold for the social welfare parameters used in the DICE-(1994,199) model. It is difficult to assess the exact reasons for this, but the result in itself shows that the choice of time discounting and elasticity of marginal utility will matter when comparing different policy scenarios.

3.4 Climatic thresholds

The next two policy categories impose different concentration and temperature limit constraints which comply to quantitative regulation forms such as quotas, targets or commands. Computationally, these are similar to the *optimal* run but with an upper limit climate constraint imposed. The first is an upper atmospheric CO_2-concentration level constraint which is set in relation to preindustrial levels. The second is an upper limit on global temperature increase compared to 1900 levels. The economic intuition behind these upper limit constraints is to consider them as threshold levels after which damages become infinitely large. For both of these constraints we see that as the limits become increasingly generous, the sustainability increase in both columns. This pattern may at first glance seem counterintuitive, but is simply a consequence of modelling assumptions. The reason for this behavior is that the production possibility set will become increasingly limited as the emissions associated with production must be constrained to correspond with stricter upper limit climate constraints. Hence, the costs of the restrains on capital gains resulting from stricter climate constraints seem to outweigh the benefits associated with reduced damages from global warming. However, compared to the BAU scenario the upper limits of $2 \times CO_2$-concentration levels and $2.5°$-temperature rise produce more sustainable results for both columns. The restrictions imposed are all binding in the optimization problem except in the case of $2.5 \times CO_2$-concentration level which therefore takes on the same value as in the *optimal* policy run.

3.5 Ambitious proposals

The last three policy runs constitute ambitious proposals which call for sharp emission reductions.

The *Gore proposal* run implies a rise in the emission control rate from 15 percent in 2010 to 90 percent in 2050, further it is assumed that participation increases from an initial 50 percent to a 100 percent by 2050. This policy run does not improve sustainability compared to BAU regardless of the choice of social welfare parameters. The reason for this is that with these climatic constraint policies, a high emission control rate reduces production possibilities to a greater extent than the benefits received from damage reduction.

The *Optimal (low-cost backstop)* policy analyzes the implications of the development of a new energy source that could replace fossil fuels at a cost that is competitive with today's technologies. This scenario corresponds to the *optimal* run except that the price for the backstop technology is reduced, which in turn lowers abatement costs. This means that more abatement can be done for the same amount of money; leading to a higher sustainabilty measure.

3.6 The Stern Review

In order to assess the results of applying the discounting assumptions adopted in the Stern Review Nordhaus uses, what he terms a dual-discounting approach. Here the emission control rates corresponding to the discounting assumptions used in the Stern Review are set as constraints on the *Optimal* policy scenario using the discounting assumptions of the DICE-2007 model. The reason for carrying out this exercise is that a meaningful comparison of policy scenarios requires that social preference structures are the same.

Even though compelling, I find a certain awkwardness to this approach. In the simulation run Nordhaus takes market preferences to be the correct proxy for social preferences. He then tries to assess the impacts of forcing emission standards corresponding to governmental preferences upon this society. However, if a government, discounting in a Stern like fashion, were to follow an optimal emission control rate the optimal emission path would be optimal only conditional upon the corresponding optimal savings rate. This means that the governments choice of emission control rate would depend on the the markets choice of saving rate and vice versa. Based on these conjectures the sustainability measure becomes difficult to interpret. However for sake of completeness the result of this policy run is presented in the last row of table 1a.

The results of a policy scenario adopting the simple discounting assumptions of the Stern Review is presented in table 1b. This policy run corresponds to the *Optimal* run in table 1a but with a utility discount rate of 0.1% and a elasticity of marginal utility of 1. This results in a sustainability measure of approximately 1731 units of social welfare. As previously pointed out this measure is not directly comparable to that of table 1a since different preference structures are applied. However, it can be noted that a lower utility discount rate implies that future well-being is valued higher, while a lower elasticity of marginal utility implies that present value of a marginal increase in consumption will rise compared to an equivalent marginal increase in the future. Therefore, as indicated by the relatively lower estimate of sustainability, the effect of lowering the elasticity of marginal utility seems to dominate the sustainability impact of a decrease in the utility discount rate.

4 Effects of uncertainty

4.1 Characteristics of uncertainty

As was shown in the previous section, the projections of sustainability within the DICE-model rely heavily upon the assumptions made regarding the estimates of social welfare parameters. The other parameter estimates are based on statistical analysis and scientific predictions regarding the evolution the relevant climatic and socioeconomic variables. Usual problems of incomplete knowledge regarding for example measurement and structural issues generate uncertainty. In this section I will address uncertainty regarding the estimated values for some of the key parameter estimates. I will restrict the analysis to the *business as usual* policy scenario and optimal policy scenarios.[17] I will further use the basic references regarding parameter uncertainty provided by Nordhaus (2007) with just a few exceptions. Nordhaus selected eight major parameters of uncertainty, including: the growth rate of total factor productivity, the rate of decarbonization, temperature sensitivity with regard to a doubling of CO_2-concentration levels, damage to output from greenhouse warming, the cost of a backstop technology, population growth, the atmospheric retention fraction of CO_2, and the total availability of fossil fuels.[18] To this list, which is presented in Table 1, I have also added uncertainty regarding the choice of utility discount rate and elasticity of marginal utility. I have chosen a normal distribution for both these model parameters, with an expected value of 1.5% and a standard deviation of 0.4% for the utility discount rate and an expected value of 2 and a standard deviation of 0.15 for the elasticity of marginal utility.[19] This means that roughly 99.7% of all parameter draws fall in the range [0.3,2.7] for the util-

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[17] Uncertainty has also been addressed within other policy scenarios. These simulations have led to similar conclusions to that of the BAU scenario and are therefore left out.

[18] In Nordhaus (2007) a technical background is given regarding the estimation processes underlying some of the most model-sensitive parameters.

[19] Expected values of 3% and 1 where used for the DICE-94 simulation in section 4.3

ity discount rate and in the range [1.55,2.45] for the elasticity of marginal utility.[20] Further, I assume that future utility is valued less than current utility and therefore discard all negative values for the utility discount rate. Considering, asymptotic population, this parameter is also drawn from the normal distribution. All values drawn above or below four standard deviations from the mean are discarded. This implies that world population will never decline bellow 1850 year levels. All other estimates with exception for the temperature sensitivity parameter are drawn from the normal probability distributions with means and variances provided in Table 1.

Table 1: Uncertainty regarding specific parameter assumptions.

Parameter	Definition	Mean	S.D.
ρ	Pure rate of time preference (time discount rate)	0.015*	0.004
η	Elasticity of marginal utility	2.0*	0.15
g_a	Growth rate of total factor productivity	0.092	0.04
g_σ	Growth rate of CO2-emission to output ratio	-0.073	0.02
$T2 \times CO_2$	Temperature sensitivity (°C) to CO_2 doubling	3.06	1.01
π_2	Damage parameter	0.0028	0.0013
P_{back}	Cost of backstop technology ($)	1170	468
$PopAsym$	Asymptotic global population	8600	1892
$CarCyc$	Transfer coefficient in carbon cycle	0.189	0.017
$Fosslim$	Total amount fossil fuels (Billion tons of carbon)	6000	1200

*The monte-carlo analysis of section 4.3 uses mean values of 3% and 1 respectively for the simulations of the DICE-(1994,1999) model.

The effect on atmospheric temperature associated with a doubling of CO_2 ($T2 \times CO_2$) is set to 3 in the DICE-model, however many estimates from models and observations yield broad and asymmetric probability distributions. Roe and Baker (2007) derive a theoretical probability distribution for temperature sensitivity which, in a satisfactory way manages to approximate, several other published probability distributions using a simple system of linear physical feedback processes. I model uncertainty regarding temperature sensitivity using the theoretical probability distribution derived in their article. Inline with an example from this article, I assume a normal feedback distribution with a mean and a standard deviation of 0.1 and leave out upper tail values exceeding three standard deviations in order to avoid catastrophic runaway feedback effects. This means that 99.9% of the temperature sensitivity values will fall in the region [1.7, 12]. The theoretical distribution derived

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[20] The utility discount rate was set as large as possible without risking the possibility of drawing negative values in the monte carlo simulation conducted in section 4.3. Concerning the elasticity of marginal utility the Gams software had problems solving the model for values above 2.7 so the standard deviation was set in order to not risk that values above 2.7 were drawn.

by Roe and Baker (2007) is generated by passing the draws from the selected feedback distribution through the equation given in Figure 1, of their article. Given the expected value and standard deviation of the feedback parameter, this generates a highly skewed distribution for temperature sensitivity with a pronounced upper tail that carries an average expected value of 3.06 and a standard deviation of 1.01.

4.2 Individual effects

In figure 1 I have outlined the effects on the sustainability criterion under the BAU scenario, when varying some of the most sensitive model parameters. Each line in the graphs represents the sensitivity of the sustainability criterion when varying that specific parameter. The upper graph shows the sensitivity of the sustainability criterion to the assumptions underlying key economic parameters, while the lower graph displays sensitivity to important climate parameters. An immediate observation when studying these two graphs is that the sustainability criterion proves to be much more sensitive to the parameter assumptions underlying the economic part of the model compared to that of the climate part. This is interesting because it means that even if we were to choose climate parameter estimates that lie far out in the tail of their respective probability distributions this would still not matter much with respect to sustainability in comparison to just a marginal change in either one of the social welfare parameters.

From the upper graph we can further see that the elasticity of marginal utility has the largest impact on sustainability in the sense that the variation in the sustainability criterion is the largest for this variable.[21] The sustainability criterion increases exponentially as this estimate increases in value. Although not shown in the figure, values below 0.9 for this parameter will generate negative values for the sustainability criterion.

As previously mentioned we cannot compare the sustainability measure for different values of η, in the sense of saying that one is more sustainable than the other. The reason for this is that social welfare is being measured using a cardinal utility function where preferences are assumed to be identical among all individuals. By definition this utility function allows only for ranking of different consumption bundles conditional on the assumed preference structure. For the exact same reason the definition of sustainability used in this paper, has to be conditioned on the assumed social preferences and cannot be compared to other potential societies having different preferences. However, the sensitivity analysis points out the importance of

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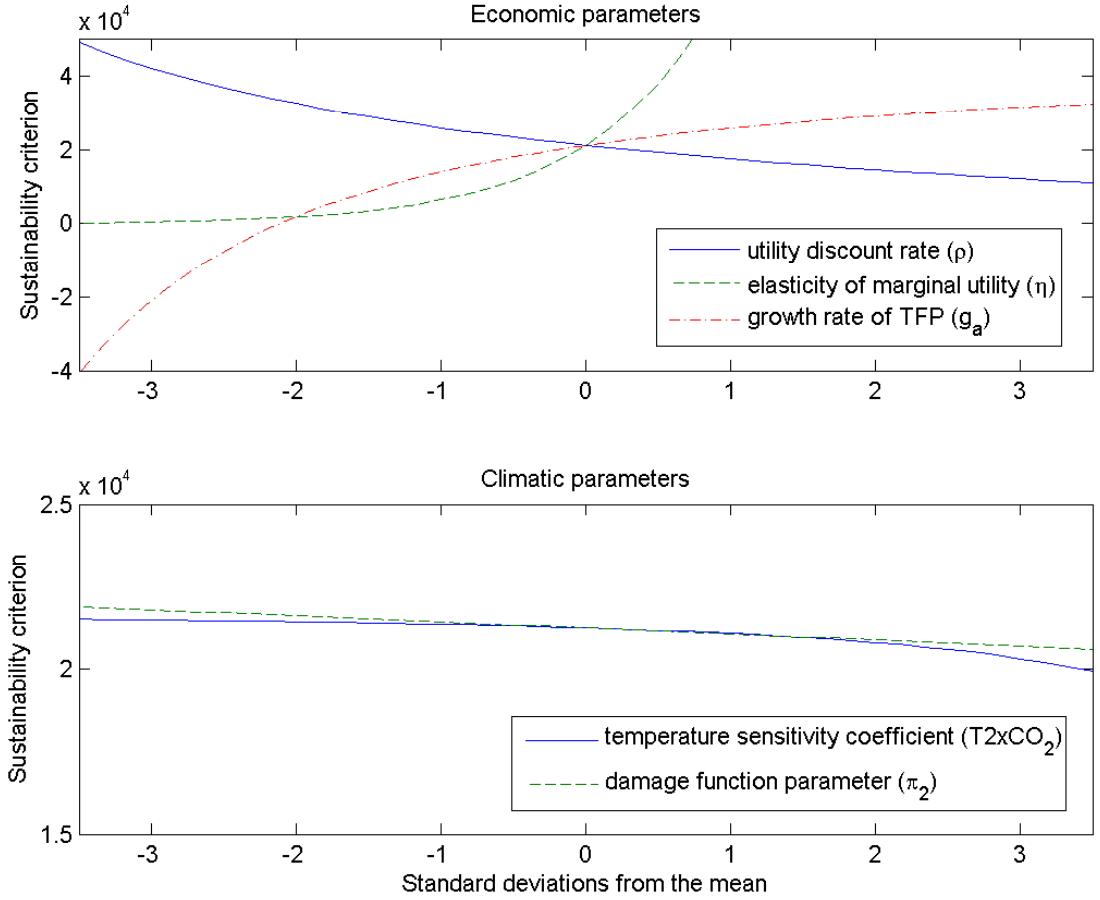
[21] The difference in sustainability values attained at 3.5 and -3.5 standard deviations from the mean.

Figure 1: Sustainability effects from varying parameter values for the *business as usual* scenario.



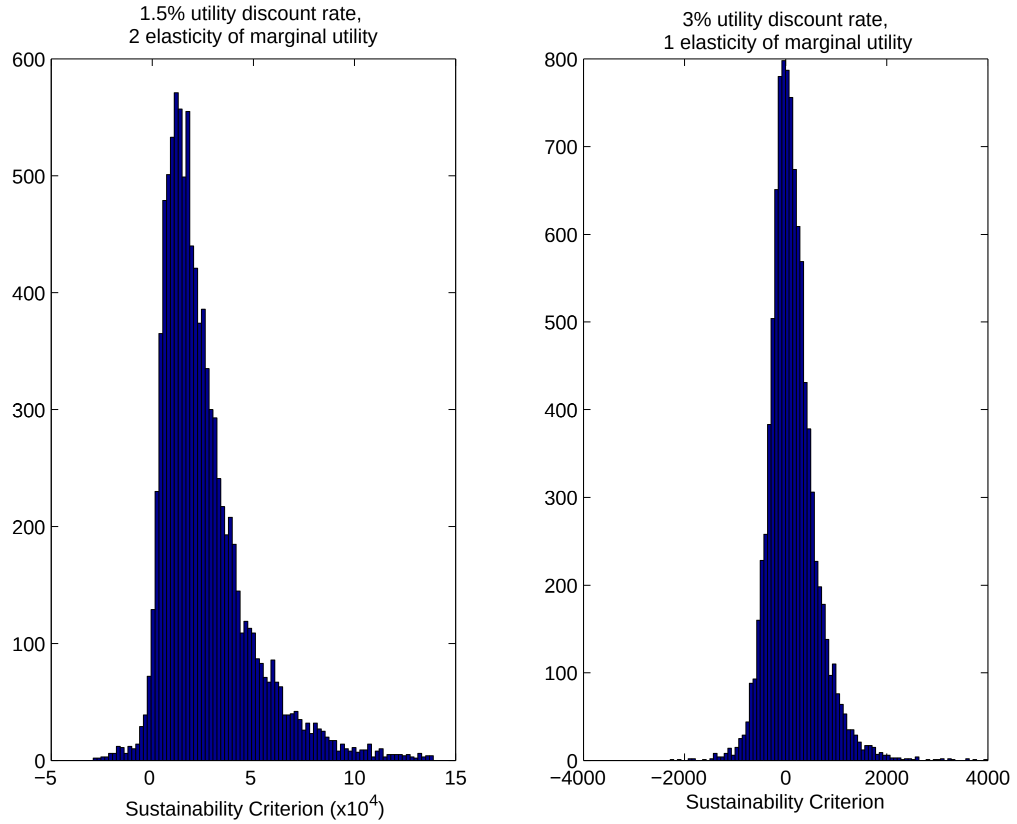
choosing the estimate of η with great care since the selection can greatly alter the model outcome with respect to sustainability.

The growth rate of total factor productivity is the second most model sensitive parameter. Judging from the variation in this parameter, the model becomes unsustainable at about -2 standard deviations from the mean, i.e. the model has about a 2.5% probability of becoming unsustainable, cetris paribus. The utility discount rate is the third most model sensitive parameter. The sustainability criterion increases in an exponential fashion when the utility discount rate approaches zero and declines slowly when the discount rate increases. The model becomes unsustainable first when the utility discount rate is roughly 6.2% i.e at about 12 standard deviations from the mean. This means that if the model predictions were to hold with certainty we could allow ourself to discount at a rate as high as 6.2% before the *productive base* we leave behind is smaller than the one we started with.

4.3 Synergistic effects

Although the above analysis provides an indication of how each specific parameter value effects the sustainability criterion, it does not capture the possibility of interacting effects amongst them. The histograms in figure 2 shows the empirical probability distributions of the sustainability criterion, for the BAU scenario under the two different discounting assumptions used in the DICE-(1994,1999) and DICE-2007 model.[22] The results were generated in a Monte Carlo simulation based on 10000 draws using the uncertainty assumptions depicted in table 1. The model was thus optimized 10000 times for each set of parameter estimates that were randomly drawn from their respective probability distributions. By recording the sustainability criterion for each set of parameter estimates the empirical probability distributions depicted in figure 2 were generated. The first of these two distributions was generated using the discounting assumptions from the DICE-2007 model.

Figure 2: Distribution of the sustainability criterion for the BAU scenario.



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[22] The mean values for ρ and η in table 1 were thus set to correspond to the DICE-(1994,1999) model.

In comparison too the DICE-(1994,1999) model these assumptions generate a histogram with a wider distribution and a more pronounced right skew. By subsequently fixating the investigated parameters, one at a time, It is found that the asymmetry of this distribution is generated due to the variation in η. When the value of η is pushed above 2, the sustainability criterion increases in an exponential fashion. This is illustrated in figure 1. In particular this generates a long right tail but it also works to spreads out the distribution as a whole.

However, an increase in η can also result in decreased sustainability. An example of this could arise when occasional negative values are drawn for the growth rate of total factor productivity.[23]

Based on these empirical probability distributions, the probability of arriving at an unsustainable path is approximately 2.85% in the DICE-2007 model and about 41.33% for the DICE-(1994,1999). These probabilities are represented by the lower tail of their respective histograms depicted in figure 2. Monte Carlo simulations were also performed for the other scenarios. These histograms had similar shapes to the ones showed in figure 2 with their respective means and medians distributed in a fashion consistent with the results displayed for the mean values used in section 3 of this paper.

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[23] For arguments and evidence regarding negative growth rates see Dasgupta *et al.* (1999) and Vouvaki and Xepapadeas (2008b).

5 Conclusions

This paper explores how sustainable development in terms of society's future production possibilities can be analyzed within an integrated assessment model of climate change. The study shows that the DICE-2007 model is production base sustainable in its current form but that the degree of sustainability is highly dependent on the values of key parameter estimates. When revising the social welfare function using estimates for the utility discount rate and elasticity of marginal utility originally put forth in the DICE-(1994,1999) model, this results in an unsustainable productive base. This finding implies that the delicate matter of adjusting the parameters of the social welfare function, is not simply solved by calibrating to an assumed rate of return on capital. Instead they should be calibrated independently, considering each parameter estimates individual effect on social welfare. Further, when assessing uncertainty regarding parameter estimates it is found that total factor productivity growth along with the social welfare parameters are those that to the greatest extent influence whether or not the model will maintain a sustainable productive base. This analysis shows that the economic parameters completely dominate important climatic parameters such as the damage parameter or temperature sensitivity coefficient, in the sense that small incremental changes in the economic parameters have a much larger effect on sustainability than corresponding large changes in the climatic parameters. This poses an interesting question as to whether the construction of the model is flawed in the sense that it does not give sufficient weight to climatic issues. Even when very extreme estimates are assumed for the climate change parameters, this still does not effect the model outcome w.r.t. sustainability as much as a slight reduction in the utility discount rate would have. As there seems to be little consensus regarding what values should be given to the parameters of the social welfare function it is therefore in my point of view, at present time, inappropriate to use integrated assessment models such as the DICE-model as policy recommendation tools. However, the model still carries a high educational value and provides an excellent tool for simulating alternative scenarios and comparing outcomes.

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Appendix

Table 2: Endogenous relationships

(A.1)	$V(t) = \sum_{\tau=t}^{T} (1+\rho)^{-(\tau-t)} U[c(\tau)] L(\tau)$
(A.2)	$U[c(\tau)] = c(\tau)^{(1-\eta)}/(1-\eta), \;\; c(\tau) = C(\tau)/L(\tau)$
(A.3)	$Q(\tau) = \Omega(\tau)[1-\Lambda(\tau)] A(\tau) K(\tau)^{\gamma} L(\tau)^{(1-\gamma)}$
(A.4)	$K(\tau+1) = K(\tau)(1-\delta)^{10} + 10I(t)$
(A.5)	$\Omega(\tau) = 1/[1 + \pi_1 T_{AT}(\tau) + \pi_2 T_{AT}(\tau)^2]$
(A.6)	$\Lambda(\tau) = \pi(\tau)\theta_1(\tau)\mu(\tau)^{\theta_2}, \;\; \pi(\tau) = \varphi(\tau)^{1-\theta_2}$
(A.7)	$Q(\tau) = C(\tau) + I(\tau)$
(A.8)	$E(\tau) = \sigma(\tau)(1-\mu(\tau))A(\tau)K(\tau)^{\gamma}L(\tau)^{(1-\gamma)} + E_{land}(\tau)$
(A.9)	$M_{AT}(\tau) = E(\tau) + \phi_{11} M_{AT}(\tau-1) + \phi_{21} M_{UP}(\tau-1)$
(A.10)	$M_{UP}(\tau) = \phi_{12} M_{AT}(\tau-1) + \phi_{22} M_{UP}(\tau-1) + \phi_{32} M_{LO}(\tau-1)$
(A.11)	$M_{LO}(\tau) = \phi_{23} M_{UP}(\tau-1) + \phi_{33} M_{LO}(\tau-1)$
(A.12)	$F(\tau) = F_{2\times co_2}\{log[M_{AT}(\tau)/M_{AT}(1750)]\} + F_{ex}(\tau)$
(A.13)	$T_{AT}(\tau) = T_{AT}(\tau-1) + \xi_1\{F(\tau) - \frac{F_{2\times co_2}}{T_{2\times co_2}} T_{AT}(\tau-1) - \xi_2[T_{AT}(\tau-1) - T_{LO}(\tau-1)]\}$
(A.14)	$T_{LO}(\tau) = T_{LO}(\tau-1) + \xi_3[T_{AT}(\tau-1) - T_{LO}(\tau-1)]$

Table 3: Exogenous relationships

(A.15)	$L(\tau) = L(t)[1 - (e^{n(\tau-t)} - 1)/e^{n(\tau-t)}] + L_{max}[(e^{n(\tau-t)} - 1)/e^{n(\tau-t)}]$
(A.16)	$A(\tau+1) = A(\tau)/(1-g_a(\tau)), \;\; g_a(\tau) = g_a(t)e^{-d(\tau-t)}$
(A.17)	$\sigma(\tau+1) = \sigma(\tau)/(1-g_{\sigma}(\tau)), \;\; g_{\sigma}(\tau) = g_{\sigma}(t)e^{-d_{sig}(\tau-t)-d_{sig2}(\tau-t)}$
(A.18)	$\theta_1(\tau) = P_{back}(t)\sigma(\tau)[(\zeta - 1 + e^{-g_{back}(\tau-t)})/(\zeta\vartheta)]$
(A.19)	$E_{land}(\tau) = E_{land}(t) * 0.9^{(\tau-t)}$
(A.20)	$CCum(t) \geq \sum_{\tau=t}^{T} E(\tau)$
(A.21)	$F_{EX}(\tau) = \begin{cases} F_{EX}(t) + 0.1(F_{EX}(t+10) - F_{EX}(t))(\tau-t), & (\tau-t) \leq 11; \\ F_{EX}(t) + \Upsilon(t), & (\tau-t) > 11; \end{cases}$

List of parameter and variables

Table 4: Endogenous variables

$V(t)$	Social welfare at initial time t
$U(\cdot)$	Utility function
$c(\tau)$	Per capita consumption
$Q(\tau)$	Output net damages and abatement (trillion 2005 U.S. dollars)
$K(\tau)$	Capital stock (trillion 2005 U.S. dollars)
$E(\tau)$	Total emission of tons CO_2 (billion tons)
$F(\tau)$	Total amount of radiative forcing (watts per square meter)
$I(\tau)$	Investment (trillion 2005 U.S. dollars)
$C(\tau)$	Consumption (trillion 2005 U.S. dollars)
$\Omega(\tau)$	Damage function
$\Lambda(\tau)$	Abatement-cost function
$\pi(\tau)$	Participation cost markup
$\mu(\tau)$	Emission-control rate
$\varphi(\tau)$	Participation rate
$M_{AT}(\tau), M_{UP}(\tau), M_{LO}(\tau)$	Mass of carbon in atmosphere, upper and lower oceans reservoirs (billion tons)
$T_{AT}(\tau), T_{LO}(\tau)$	Global mean atmospheric and lower ocean temperatures (^{o}C increase from 1900)

Table 5: Exogenous variables and parameters

τ, t, T	Variable, initial and final time periods respectively (decades)
$L(\tau)$	Population (millions)
ρ	Pure rate of social time preference
η	Elasticity of marginal utility
γ	Elasticity of output w.r.t. capital
δ	Depreciation rate of capital
$A(\tau)$	Total factor productivity
π_1, π_2	Parameters of damage function
$\theta_1(\tau)$	Adjusted cost for backstop technology
θ_2	Parameter of abatement-cost function
$\sigma(\tau)$	Emission to output ratio
$E_{land}(\tau)$	Emission of carbon from land use (billion tons)
$\phi_{11}, \phi_{12}, \phi_{21}, \phi_{22}, \phi_{23}, \phi_{32}, \phi_{33}$	Parameters of carbon cycle (flows per period)
ξ_1, ξ_2, ξ_3	Parameters of temperature equations (flows per period)
$F_{2\times co_2}$	Estimated forcings of equilibrium to a doubling of CO_2 mass
$T_{2\times co_2}$	Equilibrium temp. sensitivity to a doubling of CO_2 mass
n	Growth rate of population (decade)
L_{max}	Asymptotic population
g_a	Growth rate for total factor productivity (decade)
g_σ	Growth rate for sigma (decade)
$\zeta, g_{back}, \vartheta$	Parameters of the abatement-cost function
P_{back}	Price of backstop technology
$CCum(\tau)$	Maximum cumulative extraction fossil fuels
$F_{EX}(\tau)$	Exogenous forcing of other greenhouse gases

Table 6: Productive Base (Nordhaus, 2007)

	$\tau = 2005$	$\tau = 2015$	
$K(\tau)$	137	*	U.S.D. trillions
$L(\tau)$	6514	7130	World pop. millions
$A(\tau)$	0.027	0.030	Initial level of total factor productivity
$\sigma(\tau)$	0.134	0.125	CO2-equivalent emissions-GNP ratio 2005
$M_{AT}(\tau)$	808.9	*	Concentration in atmosphere (GtC)
$M_{UP}(\tau)$	1255	*	Concentration in upper strata (GtC)
$M_{LO}(\tau)$	18365	*	Concentration in lower strata (GtC)
$T_{AT}(\tau)$	.0068	*	2000 atmospheric temp change (C)from 1900
$T_{LO}(\tau)$	.7307	*	2000 lower strat. temp change (C) from 1900
$E_{land}(\tau)$	11	9.9	Carbon emissions from land (GtC)
$F_{EX}(\tau)$	-0.060	-0.024	Exogenous forcing for other greenhouse gases
$CCum(\tau)$	6000	5914	Maximum cumulative extraction fossil fuels
$g_a(\tau)$	0.092	0.091	Growth rate for total factor productivity
$g_\sigma(\tau)$	-.0730	-0.071	Growth rate for sigma
$\theta_1(\tau)$	0.056	*	Adjusted cost for backstop technology

* Values are dependent on the choice of policy scenario