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Managing renewable resources facing the risk of regime shifts*

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Abstract

Resource management has to take account of the possibility of regime shifts in the ecological system that provides the resource. Regime shifts are uncertain and lead to structural changes in the system dynamics, lowering the productivity of the resource. Optimal management is driven by two considerations. First, it becomes more precautionary in case a higher stock of the renewable resource decreases the hazard rate of a regime shift. Second, it will either become more precautionary or more aggressive depending on the adjustment process towards the new steady state after the regime shift. This is essentially a consumption smoothing argument and the outcome depends on the concavity of the welfare function. In conclusion, facing the risk of regime shifts, optimal management is ambiguous but it is always precautionary if the marginal hazard rate of the regime shift is sufficiently high.

JEL classification: Q22, C61, D81

Keywords: Regime shifts, tipping, uncertainty, renewable resources, precaution

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1 Introduction

Many resources can be seen as goods and services that are provided by an ecosystem. In order to manage these resources properly, consideration of the ecosystem is important. In ecology the term regime shift was introduced for large, abrupt and persistent changes in structure and functioning of an ecosystem (Biggs et al. (2012)), lowering the productivity of the resources. For example, lakes may shift from a clear to a turbid state (Scheffer (1997), Carpenter (2003)), thereby affecting water quality, fish populations and recreation. Coral reefs may shift from a coral dominated state to an algae dominated state (Hughes et al. (2003)), thereby affecting fish populations and aesthetics. At a larger scale, the climate system may shift to a different state (Stern (2007), Lenton et al. (2008)), thereby affecting precipitation patterns and agricultural productivity. The abrupt change usually comes as a surprise because the underlying system dynamics is complex and not well understood. The system has different domains of attraction (regimes) with different characteristics but it can usually not be predicted when the system will tip from one regime to the other. Optimal management of resources therefore has to take account of uncertainty and the possibility of structural changes in the ecosystem.

The structural change in the ecosystem affects the growth function of a renewable resource. A regime shift can be modelled as a downward shock to one of the parameters of the growth function, such as the carrying capacity or the growth rate in a fishery. The idea is, for example, that if a coral reef breaks down, this has an effect on the habitat and the breeding facilities of a fish species which shifts down the carrying capacity of that fishery. The uncertain event of tipping to another regime can be modelled with a hazard rate or, equivalently, with the probability of surviving in the current regime (e.g., Kamien and Schwartz (1971), Cropper (1976), Reed (1988), Tsur and Zemel (1996)). The hazard rate will be lower for a higher stock of the resource, because the ecosystem becomes less vulnerable. This will give a precautionary incentive and decrease the exploitation of the fishery. On the other hand, if the ecosystem and therefore the resource totally collapses, the hazard rate augments the discount rate which will increase the exploitation of the fishery. In such a case, the net effect on optimal management is ambiguous. However, Polasky et al. (2011) show that if the resource does not collapse but shifts to a regime with a lower but positive carrying capacity, optimal management is always precautionary. Particularly, in case of a constant exogenous hazard rate it is optimal to wait until the event occurs and to adjust instantaneously to the lower steady-state stock of the new regime. It follows that an stock-dependent endogenous hazard rate implies precautionary behaviour, in the sense of less exploitation and a higher targeted steady-state stock before the event occurs.

This result is based on a standard fishery with a fixed price for every unit of harvest so that the marginal value of the stock is equal to the price when adjusting to the lower steady-state stock. However, when the utility of harvest is concave, another incentive for optimal management results. It depends on the adjustment path after the event whether it is better to prepare for that with more or with less exploitation before the event. This is essentially a consumption smoothing argument. It follows that a potential regime shift has a precautionary effect and an ambiguous effect on optimal management before the event occurs. A trade-off occurs if it is better to prepare for the event with more exploitation while at the same time risk can be averted with less exploitation. It will be shown that the net effect is precautionary if the marginal hazard rate of the regime shift is sufficiently high. The ambiguity was also found numerically, in a similar discrete-time model, by Ren and Polasky (2014). Our paper extends this work and characterizes pricisely what happens in the standard renewable-resource model with a concave utility function.

The literature on optimal management facing the risk of regime shifts is rapidly growing, especially with regards to potential climate change (e.g., Gjerde et al. (1999), Keller et al. (2004), Naevdal (2006), Lemoine and Traeger (2014)) but also on more general issues (e.g., Brozovic and Schlenker (2011)). The paper that comes closest to our paper is Tsur and Zemel (1998). They introduce a loss function (which is a function of the stock of pollution in their case) as a consequence of the regime shift. They show that optimal management is precautionary, assuming that this loss function is non-decreasing. We will show that this assumption does not generally hold in a standard renewable-resource model with a concave utility function and that optimal management is therefore not always precautionary. To fix ideas, we focus our discussion on harvesting a fishery facing a potential shock to the carrying capacity, but the analysis is more generally applicable to renewable resources that are subject to potential regime shifts.

Section 2 presents the fishery model and introduces the hazard rate and the shock to the carrying capacity representing the regime shift. Section 3 considers optimal management of this fishery with potential regime shifts and a concave utility function. In section 4 conditions for precautionary behaviour are derived. Concluding remarks can be found in section 5.

2 Fishery with potential regime shifts

The objective of a standard fishery is to maximize the present value of the revenue from harvesting h, that is

$$\max_{h(.)} \int_0^\infty e^{-rt} U(h(t)) dt, \tag{1}$$

where U is the revenue from harvesting h and r is the discount rate, subject to the dynamics of the fish stock S given by

$$\dot{S}(t) = G(S(t)) - h(t), G(S) \equiv gS\left(1 - \frac{S}{K}\right), S(0) = S_0, \tag{2}$$

where g is the growth rate and K is the carrying capacity of the logistic growth function G, and S_0 is the initial fish stock.

It is textbook knowledge (e.g., Clark (1990)) that it is optimal in the linear fishery, with $U(h) = ph$ and a fixed price p, to move as quickly as possible to the steady-state fish stock S^* that is characterized by the golden rule $G'(S^*) = r$. This implies a moratorium on harvesting for $S_0 < S^*$ and maximal harvesting for $S_0 > S^*$. In the non-linear fishery, where the revenues are given by a concave utility function $U(h)$, the harvesting path $h(t)$ follows the stable manifold towards the saddle-point-stable steady state S^*.

Polasky et al. (2011) model a regime shift as a shock to the carrying capacity of the fishery that jumps down from K_1 to K_2 at some point in time τ, so that the dynamic constraint becomes

$$\dot{S}(t) = G_1(S(t)) - h(t), 0 \leq t < \tau, S(0) = S_0, \tag{3}$$

$$\dot{S}(t) = G_2(S(t)) - h(t), t \geqq \tau, \tag{4}$$

where G_i is the logistic growth function with carrying capacity K_i, $i = 1, 2$.

The time τ of the shock is uncertain. This is modelled by means of the hazard rate λ. If λ is constant, the probability density function for the time τ is the exponential $\lambda e^{-\lambda t}$, which yields the cumulative density function $1 - e^{-\lambda t}$. It follows that the survival probability up to time t is given by $e^{-\lambda t}$. The mean of the exponential distribution is $1/\lambda$, which is the expected time until the shock occurs. A high λ means a high probability that the shock will occur soon. One may also say that $1/\lambda$ is an indicator for the resilience of the ecosystem. The formal definition of the hazard rate is

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{P[\tau \in (t, t + \Delta t) \mid \tau \notin (0, t)]}{\Delta t}, \tag{5}$$

where P denotes probability. It is easy to check that the exponential distribution $\lambda e^{-\lambda t}$ indeed yields the constant hazard rate λ. The definition implies that $\lambda(t)\Delta t$ approximates the probability that the shock occurs between t and $t + \Delta t$, given that it has not occurred up to time t. In order to capture the idea that the probability depends on the exploitation behaviour and the stock of the fish, the hazard rate may be stock dependent, that is $\lambda(S)$ with $\lambda'(S) < 0$.

Polasky et al. (2011) derive for the linear fishery the golden rule that characterizes the targeted steady-state fish stock before the event occurs. They show that a constant exogenous hazard rate λ does not change the golden rule. It is optimal to wait until the event occurs and to adjust instantaneously to the lower steady-state fish stock of the new regime. However, a stock-dependent endogenous hazard rate $\lambda(S)$ changes the golden rule and leads to precautionary behaviour, in the sense of less exploitation and a higher targeted steady-state fish stock before the event occurs.

Ren and Polasky (2014) put forward that this may only hold in the linear fishery with instantaneous adjustment. In a similar discrete-time model, they show numerically that for a concave utility function both precautionary and more aggressive behaviour can occur before the event. In this paper we will use the same concave utility function of harvest and pin down in the standard fishery model what happens and what drives the ambiguous result. Stock-dependence of the hazard rate still leads to precaution but a consumption smoothing argument may drive behaviour the other way. We take in the objective (1) of the fishery $U(h) = \frac{h^{1-\gamma}}{1-\gamma}, \gamma > 0, \gamma \neq 1$ and $U(h) = \ln h, \gamma = 1$, where the inverse $\gamma^{-1} = \frac{-U'(h)}{hU''(h)}$ of the parameter γ denotes the elasticity of intertemporal substitution. The analysis of a potential regime shift in the fishery becomes more complicated than in the linear case but can, to a large extent, still be done analytically, which allows conclusions in terms of the parameter γ.

3 Optimal management

The problem is solved backwards in time. The time period is split into a time period *after* the shock and one *before* the shock. First the time period after the shock is solved. This is a standard stationary fishery problem which yields an optimal after-shock value of the fishery as a function of the stock of fish $S(\tau)$ at the tipping point. Then the time period before the shock is solved which is a stochastic fishery problem, because the time of tipping τ is uncertain. The uncertainty is modelled by means of a hazard rate. As was shown by Reed (1988), this is convenient because the stochastic optimal management problem turns into a deterministic one, with the hazard rate as an extra variable, as will be seen below.

3.1 After the regime shift

The current-value Hamilton-Jacobi-Bellman equation of dynamic programming for the value function of the fishery V_2, after tipping has occurred, is given by

$$0 = \max_{h_2}[U(h_2) - rV_2(S) + V_2'(S)(G_2(S) - h_2)], \tag{6}$$

with optimality condition

$$U'(h_2) = V_2'(S), \tag{7}$$

so that the after-shock value function of the fishery V_2 becomes

$$V_2(S) = \frac{U(h_2(S)) + U'(h_2(S))[G_2(S) - h_2(S)]}{r}. \tag{8}$$

Differentiating (6) with respect to S, using (7), yields an ordinary differential equation for the harvest after the shock h_2 as a function of S:

$$[G_2(S) - h_2(S)]h_2'(S) = \gamma^{-1}[G_2'(S) - r]h_2(S). \tag{9}$$

This determines the optimal harvesting policy $h_2(S)$. The differential equation (9) can also be written as a system of differential equations in h_2 and S as functions of time:

$$\begin{aligned} \dot{h}_2(t) &= \gamma^{-1}[G_2'(S(t)) - r]h_2(t), \\ \dot{S}(t) &= G_2(S(t)) - h_2(t), S(\tau) = S_\tau. \end{aligned} \tag{10}$$

The steady state $(G_2(S_2), S_2)$ of this system is given by the golden rule of a standard fishery $G_2'(S_2) = r$. The harvesting path $h_2(t)$ follows the stable manifold towards the saddle-point-stable steady state S_2 (see also Clark (1990)).

3.2 Before the regime shift

The objective of the fishery before the shift has occurred is to maximize the expected present value of the revenue from harvesting h_1, that is

$$\max_{h_1(.)} E\{\int_0^{\tau} e^{-rt}U(h_1(t))dt + e^{-r\tau}V_2(S(\tau))\}, \tag{11}$$

where E denotes expectation, subject to the dynamics of the fish stock S given by (3), where τ is the stochastic variable. The probability that the shock occurs between t and $t + \Delta t$ can be approximated by $\lambda(S(t))\Delta t$. Using this, the current-value Hamilton-Jacobi-Bellman equation of dynamic programming for the value function of the fishery V_1, before tipping has occurred, can be derived from first principles (see Polasky et al. (2011)). This results in:

$$0 = \max_{h_1}[U(h_1) + \lambda(S)(V_2(S) - V_1(S)) - rV_1(S) + V_1'(S)(G_1(S) - h_1)]. \tag{12}$$

Note that the stochastic optimal management problem (11) has been transformed into a deterministic problem with the hazard rate $\lambda(S)$ as an extra variable. The stochastic finite-horizon problem (11) has turned into a HJB equation for an infinite-horizon deterministic problem. This property is an important reason for using the hazard rate to model uncertainty (Reed (1988), Tsur and Zemel (1996)). Note also that if the hazard rate $\lambda(S) = 0$, the standard fishery model with logistic growth function $G_1(S)$ results. We will refer to this as the *naive* solution because the potential regime shift is ignored. The steady state $(G_1(S^*), S^*)$ of the system describing the naive solution is given by the golden rule of a standard fishery $G_1'(S^*) = r$.

The optimality condition before tipping becomes

$$U'(h_1) = V_1'(S), \tag{13}$$

so that the value function of the fishery V_1 becomes

$$V_1(S) = \frac{U(h_1(S)) + \lambda(S)V_2(S) + U'(h_1(S))[G_1(S) - h_1(S)]}{r + \lambda(S)}. \tag{14}$$

Differentiating (12) with respect to S, using (13), yields an ordinary differential equation for the harvest before the shock h_1 as a function of S:

$$\begin{aligned}[G_1(S) - h_1(S)]h_1'(S) = \gamma^{-1}\Bigg[G_1'(S) - r - \lambda(S)\left(1 - \frac{V_2'(S)}{U'(h_1(S))}\right) \\ - \lambda'(S)\left(\frac{V_1(S) - V_2(S)}{U'(h_1(S))}\right)\Bigg]h_1(S).\end{aligned} \tag{15}$$

This determines the optimal harvesting policy $h_1(S)$. The differential equation (15) can also be written as a system of differential equations in h_1 and S as functions of time:

$$\begin{aligned}\dot{h}_1(t) &= \gamma^{-1}\Bigg[G_1'(S(t)) - r - \lambda(S(t))\left(1 - \frac{V_2'(S(t))}{U'(h_1(t))}\right) \\ &\qquad - \lambda'(S(t))\left(\frac{V_1(S(t)) - V_2(S(t))}{U'(h_1(t))}\right)\Bigg]h_1(t), \\ \dot{S}(t) &= G_1(S(t)) - h_1(t), S(0) = S_0.\end{aligned} \tag{16}$$

The steady state $(G_1(S_1), S_1)$ of this system is given by the golden rule:

$$\begin{aligned}G_1'(S_1) &= r + \lambda(S_1)\left(1 - \frac{V_2'(S_1)}{U'(G_1(S_1))}\right) + \lambda'(S_1)\left(\frac{V_1(S_1) - V_2(S_1)}{U'(G_1(S_1))}\right) = \\ &\quad r + \lambda(S_1)\left(1 - \frac{U'(h_2(S_1))}{U'(G_1(S_1))}\right) + \frac{\lambda'(S_1)}{r + \lambda(S_1)}\left(\frac{U(G_1(S_1)) - rV_2(S_1)}{U'(G_1(S_1))}\right).\end{aligned} \tag{17}$$

It is clear that the third term in (17) is negative because $\lambda'(S_1) < 0$ and because $V_2(S_1)$ is smaller than the optimal steady-state value $U(G_1(S_1))/r$. This pushes up the steady state S_1 as compared to the naive steady state S^*. This is simply the result of the assumption that the hazard rate λ decreases with the fish stock S. The golden rule (17) is basically the same as in (Polasky et al. (2011)) but in that paper the second term disappears because the marginal revenues are equal to the marginal values and these are both equal to the price p. This is the consequence of assuming a linear fishery and leads to the conclusion that optimal management is precautionary. However, in case of a concave utility function the second term in the golden rule (17) is either negative or positive depending on whether the optimal harvesting policy $h_2(S)$ starts below or above the level of natural growth $G_1(S)$ in the steady state S_1. This in turn depends on the optimal adjustment policy to the steady state S_2 after tipping. The marginal revenue may jump up or down at the tipping point. If the second term in the golden rule (17) is negative, the steady state S_1 is pushed up as compared to the naive steady state S^* and if it is positive, the steady state S_1 is pushed down. This is essentially a consumption smoothing argument. It follows that the precautionary argument induced by the third term in the golden rule (17) is either enhanced or mitigated or even turned around. The net result depends on the degree of concavity of the utility function (i.e., on the parameter γ which is in fact the inverse of the elasticity of intertemporal substitution) and on the marginal hazard rate. We will look at this in more detail in the next section.

In a similar model on pollution control Tsur and Zemel (1998) conclude that a potential shock to welfare induces precautionary behaviour, assuming that both the hazard rate and the penalty inflicted by the event are increasing functions of the stock of pollution. In our model the penalty is $V_1(S) - V_2(S)$ and the equivalent assumption is that both the hazard rate and the penalty are decreasing functions of the stock of fish. Indeed, if $V_1'(S) - V_2'(S) < 0$ it follows that $U'(G_1(S_1)) - U'(h_2(S_1)) < 0$, so that the second term in the golden rule (17) becomes negative and precaution is enhanced. However, we will show in the next section that the penalty function in the fishery is not necessarily decreasing, so that the result may go both ways.

4 Precaution?

Optimal management is always precautionary if the second term in the golden rule (17) is smaller or equal to zero, because the third term is negative. An ambiguous result occurs if the second term is positive. First we will focus on the case of a constant exogenous hazard rate λ, so that the third term disappears.

4.1 Exogenous hazard rate

The golden rule (17) becomes

$$G_1'(S_1) = r + \lambda\left(1 - \frac{U'(h_2(S_1))}{U'(G_1(S_1))}\right). \tag{18}$$

There are two possibilities. Either the second term on the right-hand side of (18) is negative, so that $S_1 > S^*$ (since $G_1'(S^*) = r$) which means that precaution and a higher targeted steady-state fish stock result, or the second term on the right-hand side of (18) is positive, so that $S_1 < S^*$ which means that increased exploitation and a lower targeted steady-state fish stock result. Since the utility function U is concave, this second term is negative if the optimal harvesting policy after tipping, $h_2(S)$, starts below the growth function $G_1(S)$ at the level of the fish stock S_1, and it is positive if the optimal harvesting policy after tipping, $h_2(S)$, starts above the growth function $G_1(S)$ at the level of the fish stock S_1. In the last case it is optimal to increase harvesting when tipping occurs and the fish stock is at the steady-state level S_1 before the shock, i.e. $h_2(S_1) > h_1(S_1) = G_1(S_1)$. This means that it is optimal to adjust quickly to the new steady-state fish stock $S_2 < S^*$, given by $G_2'(S_2) = r$. It is to be expected that in this case it is optimal before the shock to target for a steady-state fish stock S_1 that lies between S^* and S_2, so that S_1 is pushed below S^*. In general we have to investigate when the harvest jumps up and when it jumps down if tipping occurs at the targeted steady state before the shock. First we will show that the harvest does not change at the tipping point in the case of linear harvesting policies.

The benchmark with linear harvesting policies occurs if the elasticity of intertemporal substitution $\gamma^{-1} = 0.5$. In this case the differential equation (9) has the linear solution

$$h_2(S) = \frac{r+g}{2} S, \tag{19}$$

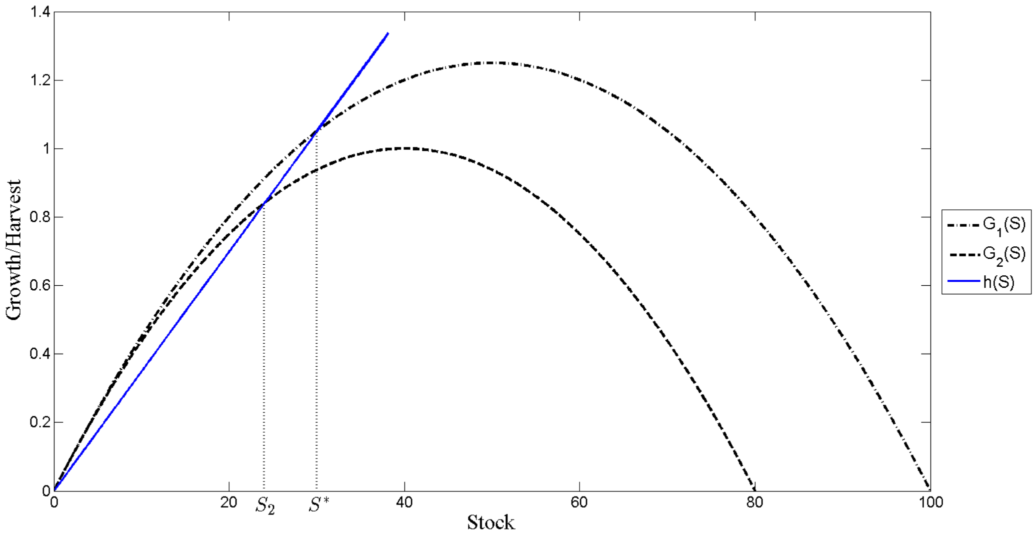
outside the singular point, of course. This harvesting policy (19) is also the optimal harvesting policy when the potential regime shift is ignored, since it does not depend on the carrying capacity K. Therefore the stable manifold (19) intersects the growth function $G_1(S)$ at the steady-state fish stock S^*. Furthermore, (19) is also the optimal harvesting policy before the shock, i.e. $h_1(S) = h_2(S)$, because the differential equation (15) reduces to the differential equation of the naive solution (equation (9) but with growth function $G_1(S)$): the third term between brackets in (15) is zero, since the harvesting level does not change when tipping occurs, and the fourth term between brackets in (15) is zero for a constant hazard rate λ. The steady states are given by

$$S_2 = \frac{K_2}{2}(1 - \frac{r}{g}), S_1 = S^* = \frac{K_1}{2}(1 - \frac{r}{g}). \tag{20}$$

Figure 1 shows the growth functions and the linear harvesting policies. We use the parameter values $g = 0.05$, $K_1 = 100$, $K_2 = 80$ and $r = 0.02$, so that $S_2 = 24$ and $S_1 = S^* = 30$.

It is interesting to note that this benchmark cannot be used in case of a downward jump in the growth rate g instead of in the carrying capacity K. In that case, the linear harvesting policies satisfy $h_2(S) < h_1(S)$ which implies precaution as we will see below. This confirms a result in Ren and Polasky (2014) where they write that the countereffect to precaution is relatively more important for a regime shift in K than in g. Of course, it is still possible to identify the benchmark where $h_2(S^*) = G_1(S^*)$ for this case but the analysis is more complicated and less transparent.

Figure 1: Linear harvesting policy



The next step is to characterize the optimal harvesting policies $h_2(S)$ for other values of the elasticity of intertemporal substitution γ^{-1}. We can find the slope of the harvesting policy $h_2'(S_2)$ in the steady-state fish stock S_2 after the shock from (9), using l'Hôpital's rule:

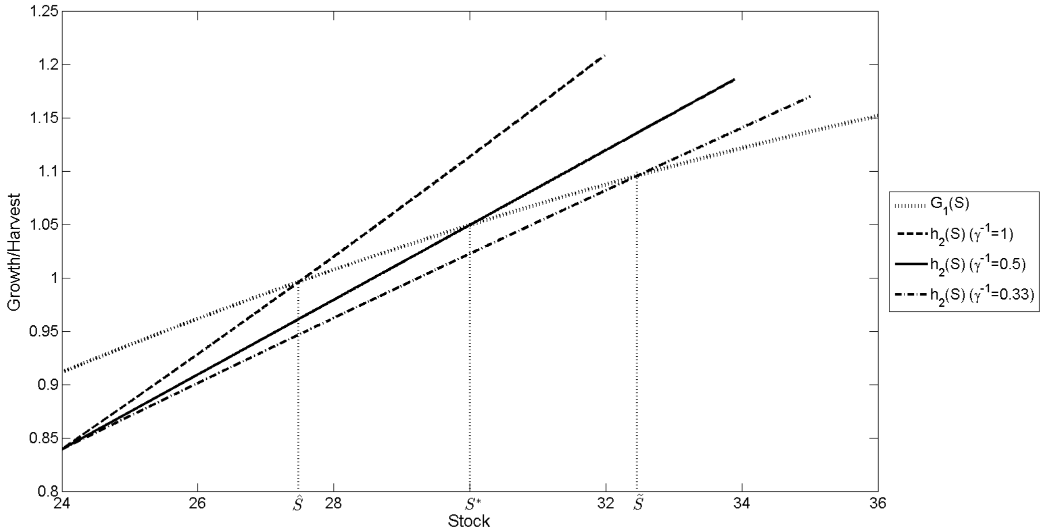
$$
\begin{aligned}
h_2'(S_2) &= \lim_{S \downarrow S_2} h_2'(S) = \lim_{S \downarrow S_2} \gamma^{-1} \frac{[G_2'(S) - r]h_2(S)}{G_2(S) - h_2(S)} \Rightarrow \\
h_2'(S_2) &= \lim_{S \downarrow S_2} \gamma^{-1} \frac{[G_2'(S) - r]h_2'(S) + G_2''(S)h_2(S)}{G_2'(S) - h_2'(S)} = \gamma^{-1} \frac{G_2''(S_2)G_2(S_2)}{r - h_2'(S_2)},
\end{aligned} \tag{21}
$$

so that $h_2'(S_2)$ is the positive root of a quadratic equation, given by

$$
\begin{aligned}
h_2'(S_2) &= 0.5\left(r + \sqrt{r^2 - 4\gamma^{-1}G_2''(S_2)G_2(S_2)}\right) = \\
&\quad 0.5\left(r + \sqrt{r^2 + 2\gamma^{-1}(g^2 - r^2)}\right).
\end{aligned} \tag{22}
$$

The elasticity of intertemporal substitution γ^{-1} determines the steepness of the optimal harvesting policy $h_2(S)$. If $\gamma^{-1} = 0.5$, the slope (22) of the harvesting policy h_2 in S_2 reduces to $(r + g)/2$, the slope of the linear harvesting policy (19). If $\gamma^{-1} > 0.5$, the slope (22) becomes larger than $(r + g)/2$ and the curve of the optimal harvesting policy $h_2(S)$ bends away above

Figure 2: After-shock harvesting policies



the linear one and intersects the growth function $G_1(S)$ at a point $\hat{S}$ below the steady-state fish stock S^*. If $\gamma^{-1} < 0.5$, the slope (22) becomes smaller than $(r+g)/2$ and the curve of the optimal harvesting policy $h_2(S)$ bends away below the linear one and intersects the growth function $G_1(S)$ at a point $\tilde{S}$ above the steady-state fish stock S^*. Figure 2 depicts these situations.

In Figure 2 we use for γ^{-1} the parameter values 0.33, 0.5 and 1. The linear optimal after-shock harvesting policy $h_2(S) = (r+g)S/2(= 0.035S)$, corresponding to $\gamma^{-1} = 0.5$, intersects the growth function $G_1(S)$ in $S^* = 30$. The other two optimal after-shock harvesting policies $h_2(S)$ are calculated numerically with the system (10), using the ODE45 solver of Matlab. First we calculate for $\gamma^{-1} = 1$ and for $\gamma^{-1} = 0.33$ the time functions $h_2(t)$ and $S(t)$, backwards in time and starting in an ϵ-neighbourhood of the steady state $(h_2(S_2), S_2) = (0.84, 24)$. The starting points are on an ϵ-circle around the steady state in the directions determined by the respective slopes (22). Then we plot the two resulting curves $h_2(S)$, to the right of $S_2 = 24$. These curves intersect the growth function $G_1(S)$ in $\hat{S}$ and $\tilde{S}$, respectively. The parameter values above yield $\hat{S} = 27.48$ and $\tilde{S} = 32.48$.

It is clear from Figure 2 what will happen. There are two possibilities. Either the steady-state fish stock S_1 before the shock lies between $\hat{S}$ and S^* or it lies between S^* and $\tilde{S}$. The first situation occurs if the elasticity of intertemporal substitution $\gamma^{-1} > 0.5$. In this case the harvesting policy $h_2(S)$ starts above the growth function $G_1(S)$ at the level of the fish stock $S_1 < S^*$ which is consistent with the golden rule (18). This is the case of increased exploitation or a lower targeted steady-state fish stock S_1 before the shock. If the elasticity of intertemporal substitution $\gamma^{-1} < 0.5$, just the opposite occurs. In this case the harvesting policy $h_2(S)$ starts below the growth function $G_1(S)$ at the level of the fish stock $S_1 > S^*$ which is consistent with the golden rule (18) as well. This is the case of precaution or a higher targeted steady-state fish stock S_1 before the shock, for the exogenous hazard rate λ. Moreover, it is easy to see from (18) that the larger the hazard rate λ, the closer the targeted steady-state fish stock S_1 lies to either $\hat{S}$ or $\tilde{S}$. To take an example, if $\lambda = 0.05$ it follows that $S_1 = 28.65$ for $\gamma^{-1} = 1$ and $S_1 = 31.53$ for $\gamma^{-1} = 0.33$, and if $\lambda = 0.1$ it follows that $S_1 = 28.23$ for $\gamma^{-1} = 1$ and $S_1 = 31.89$ for $\gamma^{-1} = 0.33$. Summarizing, we have the following result, using equation (18):

Proposition 1: For an exogenous hazard rate λ, which reflects the possibility of a regime shift, optimal management of the fishery is precautionary, if the elasticity of intertemporal substitution $\gamma^{-1} < 0.5$, and leads to increased exploitation, if the elasticity of intertemporal substitution $\gamma^{-1} > 0.5$. A higher hazard rate λ strengthens this effect and increases or decreases, respectively, the targeted steady-state fish stock S_1 before the shock. Formally:

$$S^* < S_1 < \tilde{S}\ ,\ \lim_{\lambda \to \infty} S_1 = \tilde{S},\ \gamma^{-1} < 0.5, \tag{23}$$

$$\hat{S}\ < S_1 < S^*,\ \lim_{\lambda \to \infty} S_1 = \hat{S},\ \gamma^{-1} > 0.5, \tag{24}$$

where $\tilde{S}$ and $\hat{S}$ denote the respective intersection points of the after-shock harvesting policies $h_2(S)$ with the growth function $G_1(S)$ (see Figure 2).

The intuition is simply that if the elasticity of intertemporal substitution is high, it is optimal to adjust quickly to the lower steady-state fish stock S_2 in the new regime and therefore also to target for a lower steady-state fish stock S_1 in the current regime. If the elasticity of intertemporal substitution is low, the opposite occurs. This was not found in Polasky et al. (2011) for a linear fishery because in that case the adjustment is instantaneous and therefore no incentive arises to target for another steady-state fish stock in the current regime than S^*. However, precautionary behaviour in Polasky et al. (2011) resulted from the negative dependence of the hazard rate λ on the fish stock S. This argument for precaution either strengthens the precaution found for an exogenous hazard rate λ here or turns the increased exploitation into precautionary behaviour. This is the subject of the next section.

4.2 Endogenous hazard rate

If the hazard rate is stock dependent, that is $\lambda(S)$ with $\lambda'(S) < 0$, the third term in the golden rule (17) is negative because $V_2(S_1)$ is smaller than the optimal steady-state value $U(G_1(S_1))/r$. We have seen in the previous section that the second term in the golden rule (17) is negative as well if the elasticity of intertemporal substitution $\gamma^{-1} < 0.5$. In that case optimal management of the fishery is clearly precautionary. However, if the elasticity of intertemporal substitution $\gamma^{-1} > 0.5$, the second term in the golden rule (17) is positive and then we have an ambiguous result. On the one hand, the targeted steady-state fish stock S_1 is driven below the naive level S^*, as we have seen in the previous section, but on the other hand, the targeted steady-state fish stock S_1 is pushed up because this lowers the probability of a regime shift. The net result depends on the parameter values. The two effects cancel out if the positive second term and the negative third term in the golden rule (17) add up to 0, so that $G_1(S_1) = G_1(S^*) = r$. This situation is characterized by the condition

$$\lambda(S^*)\left(1 - \frac{U'(h_2(S^*))}{U'(G_1(S^*))}\right) + \frac{\lambda'(S^*)}{r + \lambda(S^*)}\left(\frac{U(G_1(S^*)) - rV_2(S^*)}{U'(G_1(S^*))}\right) = 0. \tag{25}$$

In the previous section with an exogenous hazard rate (so that $\lambda'(S) = 0$) we calculated that for $\lambda = 0.1$ and $\gamma^{-1} = 1$ the targeted steady-state fish stock S_1 becomes 28.23 which lies below $S^* = 30$. It is clear from (17) that a sufficiently steep endogenous stock-dependent hazard rate $\lambda(S)$ can push S_1 up to S^*. In order to analyse this more precisely, we postulate the following hazard-rate function:

$$\lambda(S) = \bar{\lambda} e^{\alpha(S^*-S)}, \lambda'(S) = -\alpha \bar{\lambda} e^{\alpha(S^*-S)}, \alpha > 0. \tag{26}$$

This function has positive values, it is decreasing in S, and it is equal to $\bar{\lambda}$ for $S = S^*$. We take $\bar{\lambda} = 0.1$. The parameter α indicates the steepness of the hazard-rate function $\lambda(S)$. Substitution of this function (26) in the condition (25) leads to the condition on α where optimal management before tipping coincides with the naive solution. However, the reason now is not that the potential regime shift is ignored or that we have a linear fishery with a constant hazard rate in which it is optimal to wait until the event occurs. The reason now is that two effects cancel out. On the one hand, there is an incentive to be precautionary in order to lower the risk of a regime shift and, on the other hand, there is an incentive to aim for a lower steady-state fish stock, looking ahead towards the adjustment path after tipping. The condition (25) yields in fact a lower bound for the parameter α with the property that for values of α above this lower bound optimal management is precautionary. It follows that precautionary behaviour results if and only if the hazard-rate function is sufficiently steep or

$$\alpha > (r + \bar{\lambda}) \left(\frac{U'(G_1(S^*)) - U'(h_2(S^*))}{U(G_1(S^*)) - rV_2(S^*)} \right). \tag{27}$$

For the parameter values that we have used up to now, we get $\alpha > 0.0656$.

Note from the condition for precaution (27) the interplay between the level of the hazard rate function (26), characterized by $\bar{\lambda}$, and the steepness of the hazard-rate function (26), characterized by α. As we have seen in the previous section (for $\gamma^{-1} = 1$), a higher level of the hazard rate pushes the targeted steady-state fish stock S_1 down and closer to S_2. We can only get precaution, i.e. $S_1 > S^*$, if the marginal hazard rate is sufficiently large.

It is interesting to investigate how the condition for precaution (27) is affected by the other parameters. It is clear that the elasticity of intertemporal substitution γ^{-1} affects $h_2(S)$ (given by (9)) and thus $V_2(S)$ (given by (8)), and thus the right-hand side of the condition (27). A lower elasticity of intertemporal substitution γ^{-1} will pull $h_2(S)$ down and it is to be expected that this will allow for a lower α to get precaution. Furthermore, the size of the shock to the carrying capacity from K_1 to K_2 affects the after-shock logistic growth function $G_2(S)$ and thus the after-shock steady-state fish stock S_2, and thus $h_2(S)$ and $V_2(S)$ and the right-hand side of the condition (27). A smaller shock to the carrying capacity will require less adjustment after tipping but it also decreases the risk of tipping. It is not clear what the total effect on the lower bound of α will be. Since we had to calculate the optimal after-shock harvesting policy $h_2(S)$ numerically, we can investigate these effects only numerically as well. Table 1 gives the results.

Table 1: Lower bound of α for different values of γ^{-1} and K_2

α	$K_2 = 80$	$K_2 = 85$	$K_2 = 90$	$K_2 = 95$
$\gamma^{-1} = 1$	0.0656	0.0666	0.0670	0.0675
$\gamma^{-1} = 0.80$	0.0520	0.0525	0.0530	0.0535
$\gamma^{-1} = 0.67$	0.0360	0.0364	0.0368	0.0372
$\gamma^{-1} = 0.57$	0.0185	0.0188	0.0190	0.0192

The effect of a lower elasticity of intertemporal substitution γ^{-1} on α is indeed confirmed. The adjustment effect becomes smaller and therefore the hazard-rate function can be less steep in order to get precaution. A smaller shock to the carrying capacity leads to a higher lower bound for α, but this effect is very small.

Summarizing, an endogenous hazard rate, in the sense that the probability of a regime shift is negatively affected by the level of the fish stock, is a precautionary force. It will push up the targeted steady-state level of the fish stock S_1 before the shock. However, whether this indeed implies that $S_1 > S^*$, where S^* is the steady-state level of the fish stock in case the possibility of a regime shift is ignored, depends on the adjustment path after tipping. If the elasticity of intertemporal substitution is low, optimal management of the fishery is always precautionary but if it is high, the result is ambiguous and precaution only results if the effect of a change in the level of the fish stock on the hazard rate is sufficiently strong. We have the following result.

Proposition 2: For an endogenous hazard rate $\lambda(S)$, with $\lambda'(S) < 0$, which reflects the possibility of a regime shift, optimal management of the fishery is precautionary, if the elasticity of intertemporal substitution $\gamma^{-1} < 0.5$. If $\gamma^{-1} > 0.5$, optimal management of the fishery is still precautionary if the condition (27) holds, where α indicates the steepness of the hazard-rate function $\lambda(S)$ according to the specification (26), and otherwise it leads to increased exploitation. Table 1 shows the sensitivity of α with respect to important underlying parameters.

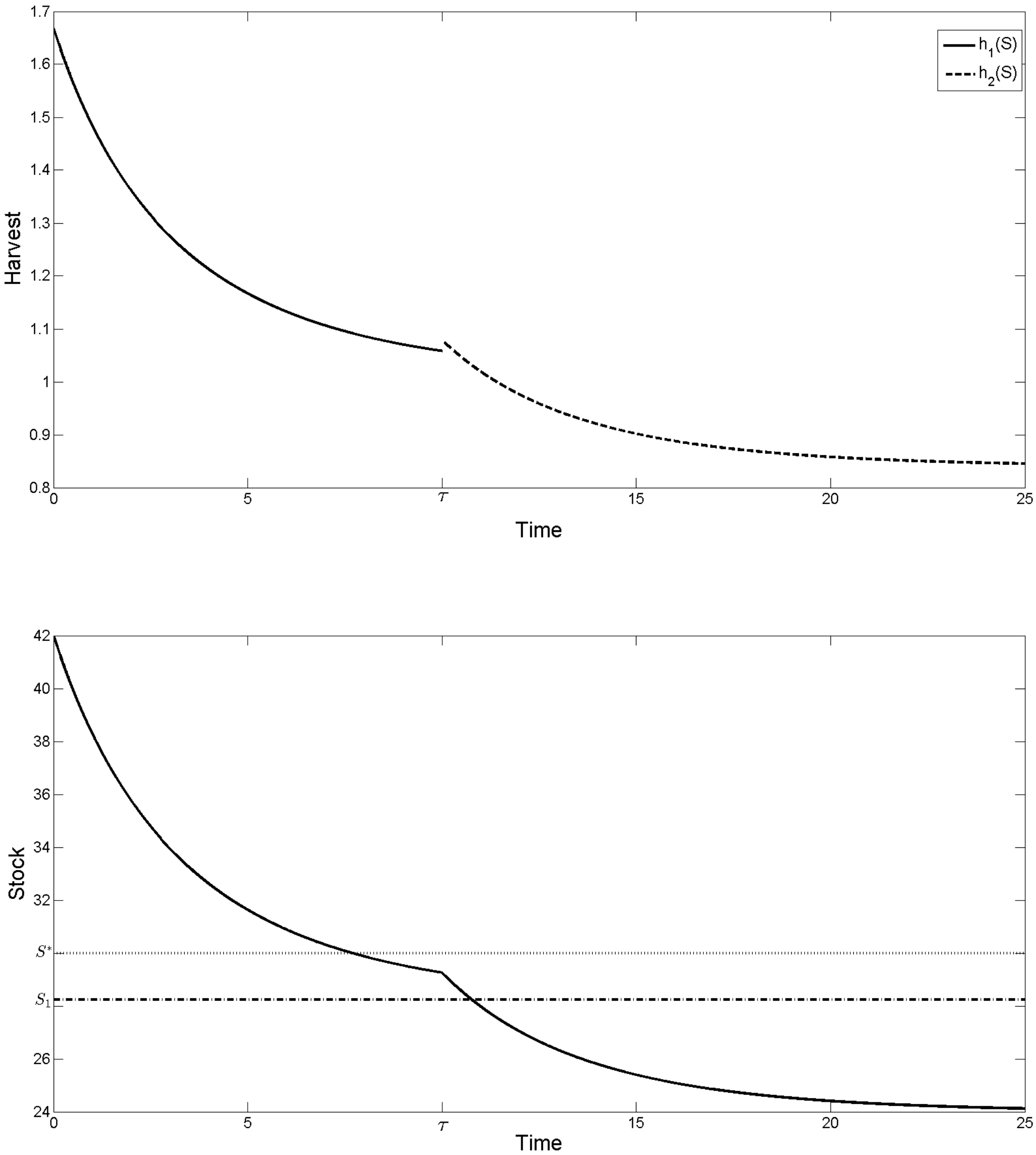
4.3 Example

We can illustrate the results with time paths for the harvest h and the fish stock S, with initial fish stock $S_0 = 42$ and using the basic parameter values $g = 0.05$, $K_1 = 100$, $K_2 = 80$, $r = 0.02$. The steady-state fish stock after the shock is $S_2 = 24$ and the steady-state fish stock ignoring the potential regime shift is $S^* = 30$. The time paths are calculated numerically with the systems (10) and (16), backwards in time, starting in an ϵ-neighbourhood of the steady states $(h_2(S_2), S_2)$ and $(h_1(S_1), S_1)$, and using the ODE45 solver of Matlab.

If $\bar{\lambda} = 0.1$ and $\alpha = 0$ the hazard rate (26) is constant and equal to 0.1. We have already seen that for the high elasticity of intertemporal substitution $\gamma^{-1} = 1$, a potential regime shift leads to increased exploitation or a lower targeted steady-state fish stock $S_1 = 28.23$ before the shock. Figure 3 shows the time paths for the harvest h and the fish stock S in this case. The targeted steady-state fish stock S_1 lies below the naive steady-state fish stock S^* and the harvest h jumps up at the tipping point τ.

For the low elasticity of intertemporal substitution $\gamma^{-1} = 0.33$ and the constant hazard

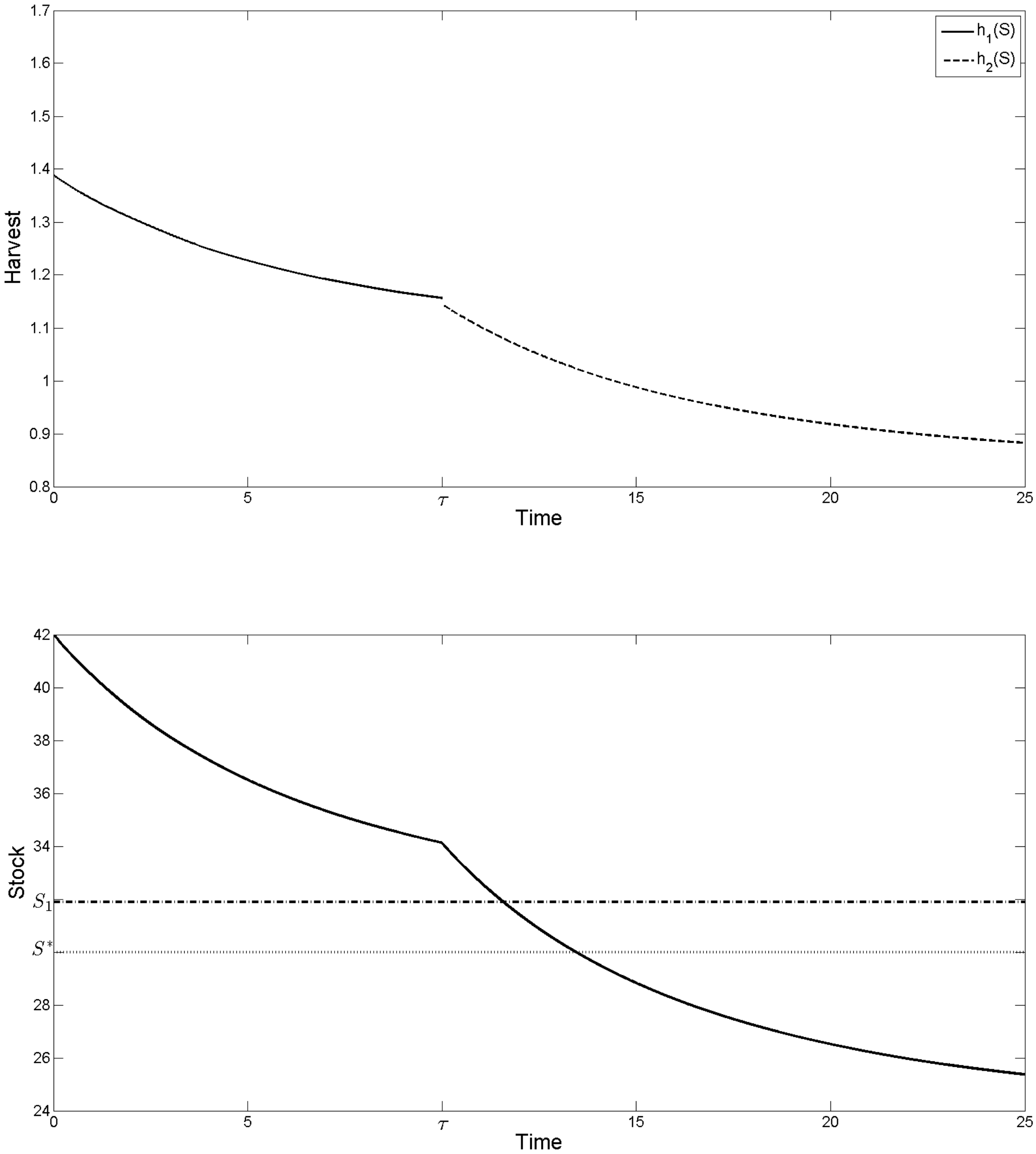
Figure 3: High elasticity of substitution with exogenous hazard rate



rate $\lambda = 0.1$, however, we get precaution or a higher targeted steady-state fish stock S_1 before the shock. The targeted steady-state fish stock is $S_1 = 31.89$, as we have seen above. Figure 4 shows the time paths for the harvest h and the fish stock S in this case. The targeted steady-state fish stock S_1 lies above the naive steady-state fish stock S^* and the harvest h jumps down at the tipping point τ.

An important result is that precaution or a higher targeted steady-state fish stock S_1

Figure 4: Low elasticity of substitution with exogenous hazard rate



before the shock remains to be optimal for high elasticities of intertemporal substitution if the hazard rate is stock-dependent and the marginal hazard rate is sufficiently high. We have seen that for $\gamma^{-1} = 1$ and $\alpha = 0.0656$ the targeted steady-state fish stock is equal to S^*. If we increase α to 0.09, then precaution results. The targeted steady-state fish stock S_1 becomes 30.85, which is larger than $S^* = 30$, but note that the harvest h jumps up now at the tipping point τ because we have not changed γ (see Figure 2). Figure 5 shows the time paths for the harvest h and the fish stock S in this case. The precautionary effect of a stock-dependent hazard rate now dominates the consumption smoothing effect and the harvest h has to jump up more than in Figure 3 in order to move optimally towards the much lower after-shock steady-state fish stock $S_2 = 24$. We have chosen the tipping point τ a bit higher than in Figures 3 and 4, because the hazard rate is lower now (see (26)) and therefore the expected time until the shock occurs is higher.

Figure 5: High elasticity of substitution with stock-dependent hazard rate

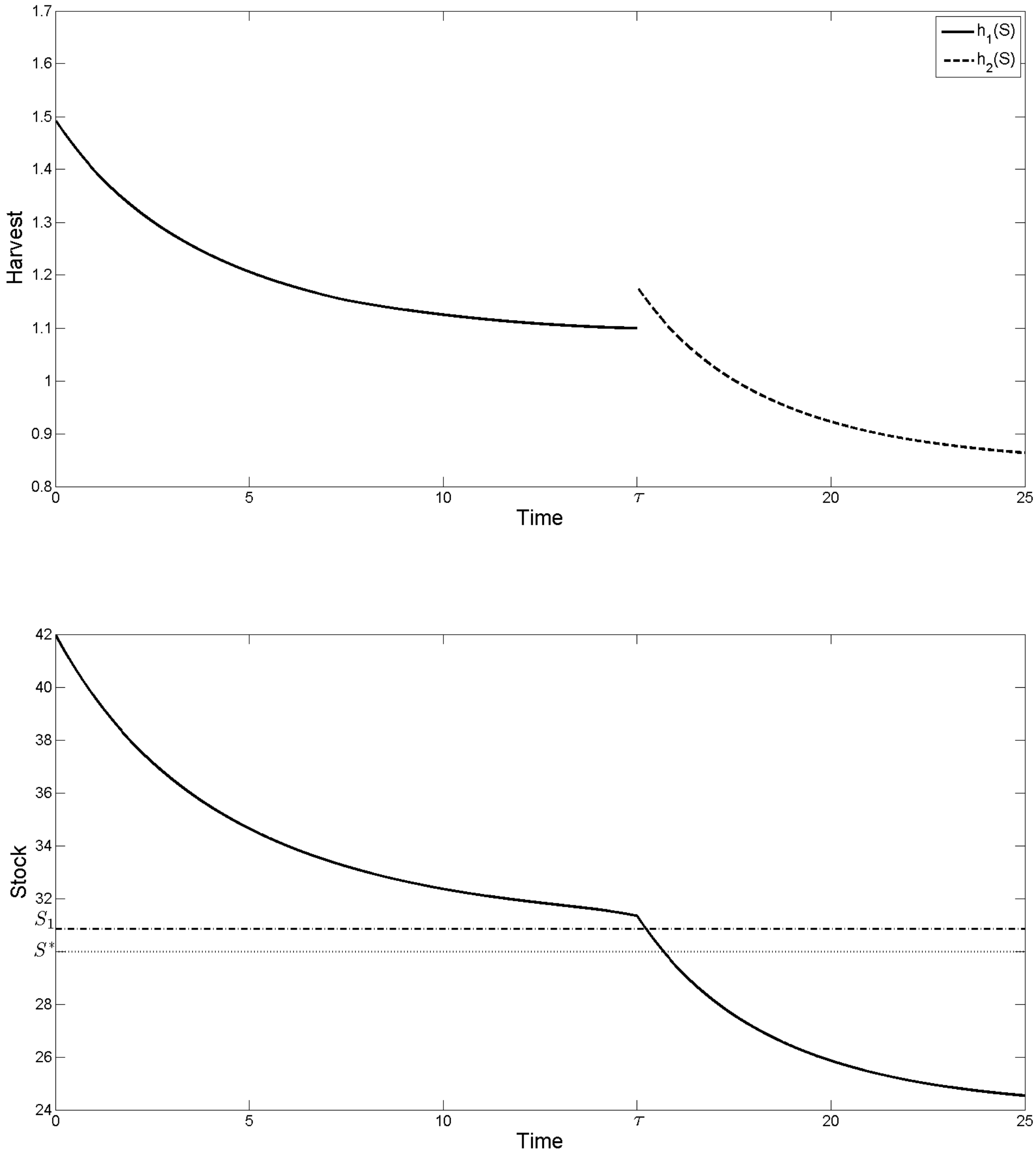
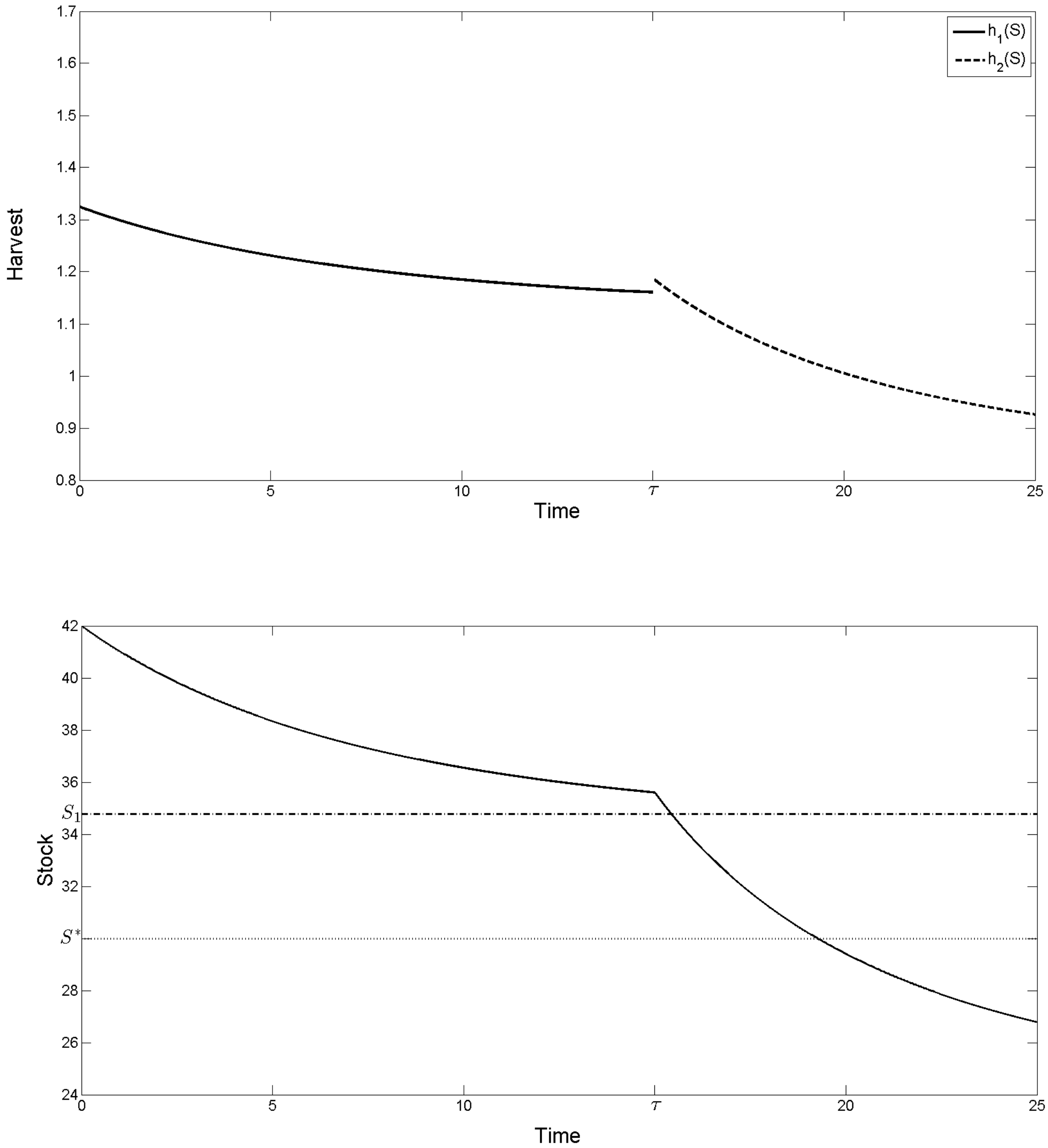


Figure 6: Low elasticity of substitution with stock-dependent hazard rate



For the low elasticity of intertemporal substitution $\gamma^{-1} = 0.33$, we already get precaution for a constant hazard rate $\lambda = 0.1$. For the stock-dependent hazard rate with $\alpha = 0.09$, the precautionary effect is enhanced. The targeted steady-state fish stock S_1 becomes 34.77 but now the harvest h jumps up again at the tipping point τ because $S_1 > \tilde{S} = 32.48$ (see Figure 2). The precautionary effect becomes so strong that even for this low elasticity of intertemporal substitution $\gamma^{-1} = 0.33$, a higher harvesting level is needed to move optimally towards the much lower after-shock steady-state fish stock $S_2 = 24$. Figure 6 shows the time paths for the harvest h and the fish stock S in this case.

Figures 3 up to 6 show the different patterns that can occur. In Figures 3 and 4 the hazard rate λ is exogenous and either increased exploitation or precaution can occur, depending on the elasticity of intertemporal substitution. In case of increased exploitation the harvest h jumps up at the tipping point τ and in case of precaution the harvest h jumps down at the tipping point τ. Figures 5 and 6 show that a stock-dependent endogenous hazard rate $\lambda(S)$ has a precautionary effect as compared to Figures 3 and 4, respectively. This precautionary effect is so strong that the net effect in Figure 5 is precautionary. The harvest h jumps up at the tipping point τ but more than in Figure 3 because the initial after-shock fish stock $S(\tau)$ is higher. In Figure 6 the precautionary effect in Figure 4 is enhanced and precaution becomes so strong now, with such a high initial after-shock fish stock $S(\tau)$, that the harvest h jumps up at the tipping point τ, in contrast to Figure 4. The harvesting policy before the shock is based on expectations and is fully determined by the system (16) and the targeted steady-state fish stock S_1. After the shock we have a standard fishery with an initial fish stock $S(\tau)$.

5 Conclusions

If resources are provided by an ecosystem that can tip to a state that is less productive, it is important to understand how optimal management of the resource is affected by this possibility. Usually it is not known when tipping will occur and how strong the shock will be, so that optimal management has to handle this type of uncertainty. It would at least be good to understand the direction that optimal management has to take, as compared to the situation where these potential regime shifts are ignored. Polasky et al. (2011) show, in a linear fishery model with potential regime shifts, that optimal management is precautionary which gives a justification for the precautionary principle. However, Ren and Polasky (2014) cast doubt on this general result and show numerically, in a similar discrete-time model, that the opposite may occur in case the utility function of harvest is concave.

The issue is that in a standard fishery model with a fixed price, adjustment to a lower steady-state fish stock is instantaneous. Therefore, for a constant hazard rate, it is optimal to wait until tipping occurs and to adjust instantaneously to the lower after-shock steady-state fish stock. As a consequence, optimal management is precautionary since the hazard rate negatively depends on the level of the fish stock. However, if the utility function of harvest is concave, the adjustment path depends on the elasticity of intertemporal substitution and an incentive may arise to target for a lower steady-state fish stock before the shock, which gives the opposite effect to precaution. Actually, the effect of such a concave objective function can go two ways. This paper shows that if the elasticity of intertemporal substitution is smaller than 0.5, precaution is enhanced but if the elasticity of intertemporal substitution is larger than 0.5, the opposite effect of increased exploitation occurs. In the last case the net effect of a potential regime shift is still precautionary if the effect of a change in the level of the fish stock on the hazard rate is sufficiently strong.

The empirical literature on the elasticity of intertemporal substitution is not conclusive (e.g., Gruber (2013)). A value of 0.5 is certainly within the range of reasonable numbers. Furthermore, little is known on the hazard functions of regime shifts. It is therefore difficult to draw a general conclusion regarding the question whether optimal management of a renewable resource facing the risk of a potential regime shift is precautionary or not. However, the precautionary effect of a stock-dependent hazard rate is there and if this effect is sufficiently strong, optimal management is always precautionary.

The results are derived for a fishery model in the context of an ecological system that is subject to a potential regime shift that affects the productivity of the fishery. However, the analysis is more generally applicable to the management of renewable resources facing the risk of tipping to another regime and a structural loss of productivity. This paper provides insight into how potential tipping drives optimal policy before tipping.

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